

Subject 13

Please, do not write on the exam paper

The original problem that Fibonacci investigated in 1202 was about how fast rabbits could reproduce. He created an imaginary set of ideal conditions under which rabbits could breed, and posed the question, "How many pairs of rabbits will there be a year from now ?"

The ideal set of conditions was as follows:

- You begin with one male rabbit and one female rabbit. These rabbits have just been born.
- A rabbit will reach sexual maturity after one month.
- The gestation period of a rabbit is one month.
- Once it has reached sexual maturity, a female rabbit will give birth every month.
- A female rabbit will always give birth to one male rabbit and one female rabbit.
- Rabbits never die.

So how many male/female rabbit pairs are there after one year (12 months) ?

QUESTIONS

We want to use a sequence in order to modelize this problem. So we label u_n the number of pairs of rabbits in the n^{th} month. So $u_1 = 1$; $u_2 = 1$

- Explain that the number of pairs for each month is given by the sum of the number of pairs in the 2 previous months or that $u_{n+2} = u_n + u_{n+1}$
- Complete the table below :

Month n°	1	2	3	4	5	6
Number of pair of rabbits (u_n)	$u_1 = 1$	$u_2 = 1$	$u_3 = 2$			

Month n°	7	8	9	10	11	12
Number of pair of rabbits (u_n)						

- How many pairs will there be after one year ?

- The Fibonacci sequence is linked to the golden proportion ($\varphi = \frac{1+\sqrt{5}}{2}$). φ is the limit of the growth

rate $\frac{u_{n+1}}{u_n}$ of the Fibonacci's terms.

- Compute some ratios $\frac{u_{n+1}}{u_n}$ to check if the given sequence is growing in the golden proportion.
- Find the 2 roots of the equation : $x^2 - x - 1 = 0$. What do you think of that ?
- Prove that φ is a root of the equation $\frac{1}{x} - x + 1 = 0$ Deduce that $\frac{1}{\varphi} = \varphi - 1$.