

## Subject 20

**Please, don't write on the exam paper.**

### Contrapositive

**definition** : the contrapositive of a logical sentence « if A then B » is « if (not B) then (not A) ».

(not A) is the logical negation of A. For example  $\text{not}(x \geq 0)$  is  $x < 0$ .

**property** : the contrapositive is equivalent to the implication. It means that they are both true or false at the same time.

We also recall that the logical negation of « A or B » is « not A and not B »

We recall that if  $n$  is an odd integer, then  $n$  can always be written  $n = 2k + 1$  ( $k$  another integer), and if  $n$  is even, then  $n = 2k$ .

1) Give the contrapositive of the following implications.

- a) if  $n > 2$  then  $n^2 > 4$ .
- b) if there is sun or moon, then there is light outside.

2) Let  $p$  and  $q$  be 2 positive integers. We want to prove the following implication :

« If  $pq$  is even then  $p$  is even or  $q$  is even. »

- a) Find the contrapositive of this implication.
- b) Prove the contrapositive.
- c) What is the conclusion ?