

Subject 9

Please do not write on the exam paper.

We suppose that, for a certain period of time, a country's population is **constant** and equals **40 million** inhabitants, among whom 10 million live in towns and 30 million live in rural zones.

We observe that the population movements are described by the following rule :
each year, 10 % of the town persons emigrate to rural zones and 30 % of the rural persons emigrate to towns.

We respectively note T_n and R_n the number of persons (in million) who live in towns and in rural zones after n years ($T_0 = 10$ and $R_0 = 30$).

a) Prove that, for all whole number n , we have :

$$T_{n+1} = 0.9T_n + 0.3R_n \quad \text{and} \quad R_{n+1} = 0.1T_n + 0.7R_n .$$

b) Prove that $T_n + R_n$ is a constant.

c) Deduce from **b)** that the sequences (T_n) and (R_n) satisfy to :

$$T_{n+1} = 0.6T_n + 12 \quad \text{and} \quad R_{n+1} = 0.6R_n + 4 .$$

d) The sequences (T_n) and (R_n) are not geometrical, but prove that (t_n) and (r_n) , defined by ($t_n = T_n - 30$ and $r_n = R_n - 10$) are geometrical sequences.

e) Express T_n and R_n in terms of n .

Then study the limits of the sequences (T_n) and (R_n) . What can you conclude ?