

Subject 19

Please, don't write on the exam paper.

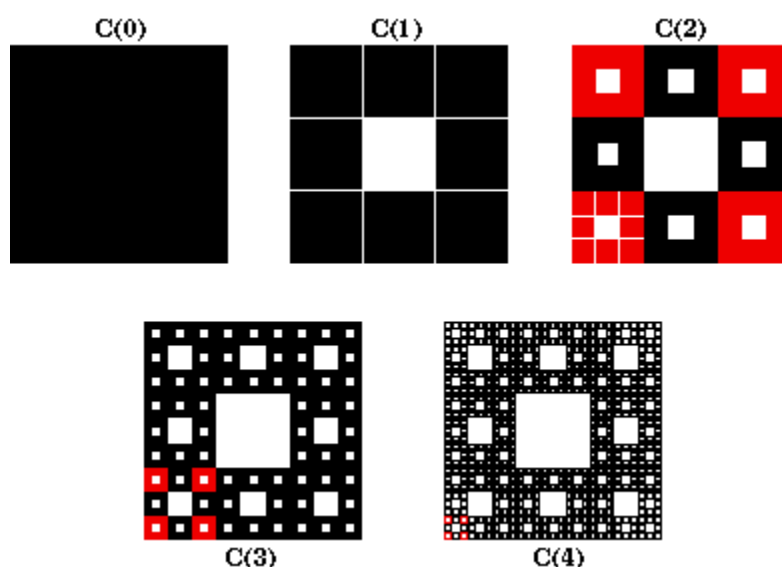
The Sierpinski Carpet

The construction of the Sierpinski carpet begins with a square of side 1.

The square is cut into 9 identical subsquares in a 3-by-3 grid, and the central subsquare is removed. The same procedure is then applied recursively to the remaining 8 subsquares.

The Sierpinski carpet at order n is denoted by $C(n)$.

We denote by A_n the area of $C(n)$.



- 1) Using the preceding image and description, explain how we obtain $C(2)$, $C(3)$ and $C(4)$.
- 2) Compute the areas A_0 , A_1 , A_2 of $C(0)$, $C(1)$, $C(2)$ respectively.
- 3) Justify that : $A_{n+1} = \frac{8}{9} A_n$.
- 4) What kind of sequence is (A_n) ? Write A_n in terms of n .
- 5) Is (A_n) an increasing or decreasing sequence?
- 6) What is the proportion of $C(n)$ vanishing at each step?
- 7) What is the smallest n_0 for which we have $A_{n_0} < 0,1 A_0$?

Optional questions

- 8) Justify that $C(n+1)$ is included in $C(n)$.
- 9) Justify that $C(n) = C(0) \cap C(1) \cap \dots \cap C(n)$ (use a recurrency if you are in S section, otherwise state it for $n=1,2,3$)
- 10) We denote by $C = \bigcap_{n \geq 0} C(n)$ the intersection of all $C(n)$. This is the limit set C of $C(n)$ as n goes to infinity, called the Sierpinski Carpet.
Make a conjecture concerning its area. Is C an empty set?