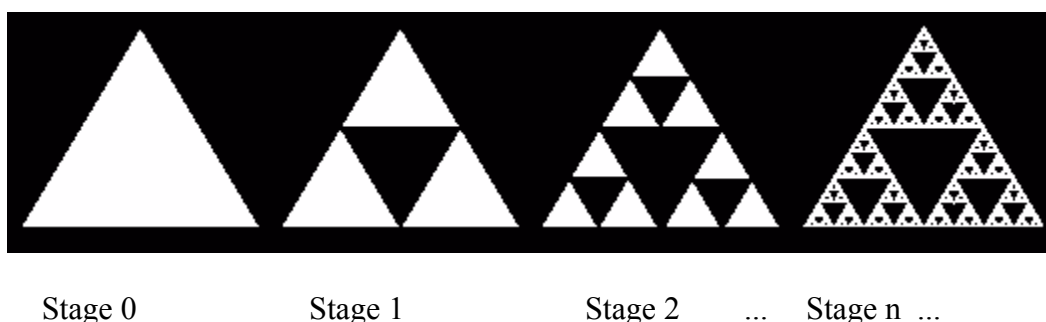


Please do not write on the exam paper.
Please do not forget to give it back at the end of the examination.

Sequences

Geometric Construction

In order to generate the Sierpinski Triangle, begin with an equilateral triangle (first figure below). Connect the midpoints of each side to form four separate triangles, and cut out the triangle in the center (second figure). For each of the three remaining triangles, perform this same act (third figure). Iterate infinitely.



Stage 0

Stage 1

Stage 2

...

Stage n ...

Questions :

A) Let R_n be the number of white (remaining) triangles at the stage n .

1. Compute R_0, R_1, R_2 .
2. Explain that for all n , $R_{n+1} = R_n \times 3$.
3. What kind of sequence is (R_n) ? Compute R_n in terms of n .
4. What is the limit of R_n as n tends to infinity?

B) We denote by T_n the area of one of the identical white triangles at stage n .

1. Compute T_0, T_1, T_2 .
2. Explain that for all n , $T_{n+1} = \frac{T_n}{4}$.
3. What kind of sequence is (T_n) ? Compute T_n in terms of n .
4. What is the limit of T_n as n tends to infinity?

C) Let us suppose that the area of the white triangle at stage 0 is 1. We denote by A_n the total area of the white triangles at stage n .

1. Compute A_0, A_1, A_2 .
2. Explain that for all n , $A_{n+1} = \frac{3}{4} \times A_n$.
3. Compute A_n in terms of n .
4. What is the limit of A_n as n tends to infinity?