Subject n°25

sequences

Please do not write on the exam paper and do not forget to give back the examination paper at the end of the test.

The Fibonacci sequence is the sequence (f_n) of integers that can be defined as follows:
$f_1 = 1$, $f_2 = 1$ and for any $n \geq 1$ $f_{n+2} = f_n + f_{n+1}$

1-a) Compute the first five terms of the Fibonacci sequence.
1-b) Prove that for any $n \geq 5$: $f_n \geq n$
1-c) What can you deduce for the convergence of (f_n)?

The Fibonacci quotient sequence is the sequence of real numbers defined as the quotient of two consecutive terms of the Fibonacci sequence.
Thus for any counting number n $q_n = \frac{f_{n+1}}{f_n}$
2-a) Prove that for any counting number n q_n exists.
2-b) Prove the recursive relation: $q_{n+1} = 1 + \frac{1}{q_n}$ for any counting number n.
2-c) Study the variations of the function f: $\begin{cases}]0;+\infty[\to \mathbb{R} \\ x \mapsto 1 + \frac{1}{x} \end{cases}$
2-d) Is (q_n) monotonous?
2-e) Let us assume that (q_n) converges towards a strictly positive limit, and denote its limit φ.
Prove that φ is the positive solution to the equation $x^2 - x - 1 = 0$
Give the exact value of φ.
(φ is called the golden ratio).