Please do not write on this paper, and do not forget to hand it back at the end.
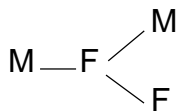
SEQUENCES

Question 1 :

The life of an ant starts from an egg : if the egg is fertilised, then the progeny will be female.

If not, the progeny will be male. Thus a male has only one mother, and a female has two parents.

Only females can lay eggs.

1) Here is the family tree of a male ant up to the second generation of ancestors.

Complete this family tree up to the fifth generation of ancestors.

2) We want to study the number of ancestors at each generation, so we define a sequence
   (u_n) starting from $u_1 = 1$ (we have only our male ant), with u_n the number of ancestors at the
   nth generation for n>1.
   Give the first five terms of this sequence.
3) Find a relation linking u_{n+2} to the two previous terms.
4) We could do the same work on the family tree of a female ant.
   What results would we get ?

Nota : ant means « fourmi »

    progeny means « progéniture »

    family tree means « arbre généalogique »

Question 2 :

We want to compare human reproduction with ant reproduction :

- draw the same kind of family tree for a human up to the fifth generation,
- count the number of ancestors at each step,
- define a sequence (v_n) to model that number : give both the explicit formula and the
  recurrence relation, as they are not too difficult to find.