

Sujet N°22

*Please do not write on the exam paper, and do not forget to give it back at the end of the test.*

**FUNCTIONS**

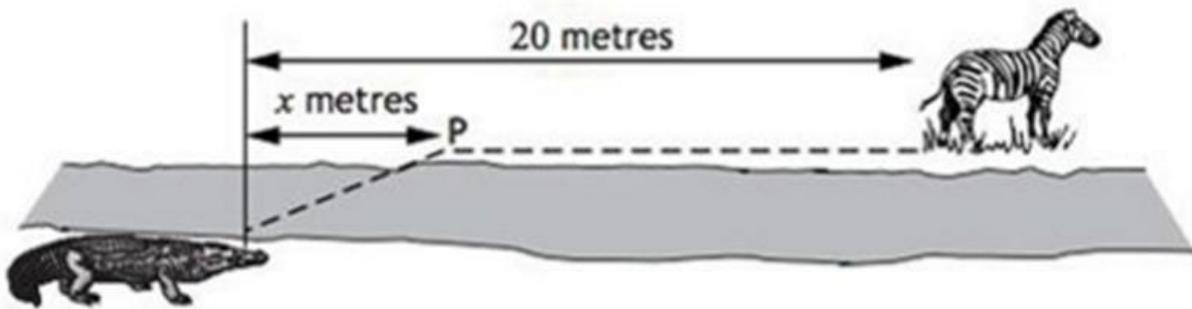
**An African problem**

A crocodile is stalking a prey located 20 metres further upstream on the opposite bank of a 6 metres wide river.

Crocodiles travel at different speeds on land and in river. The speed in river is 2 meters per second and the speed on land is 2.5 meters per second.

We recall the formula relating speed, distance and time:  $v = \frac{d}{t}$ .

The time taken by the crocodile to reach its prey can be minimized if it swims to a particular point P. This point P is  $x$  metres upstream on the other side of the river as shown in the diagram below.



In a first step, we admit that the time taken,  $T$ , measured in seconds, is given by:

$$T(x) = 0.5\sqrt{36 + x^2} + 0.4(20 - x)$$

**1.a.** Calculate the time taken if the crocodile swims only.

**b.** Calculate the time taken if the crocodile swims the shortest distance possible.

**2.** Between these two extremes there is one value of  $x$  which minimizes the time taken.

Using your calculator, try to find out an approximate value for  $x$  and hence the minimum possible time.

**3.** Prove the formula giving  $T$  with respect to  $x$ .

**4.** By studying the variations of the function  $T$ , prove the result of question 2.

**Vocabulary:**

to stalk: traquer

prey: proie

upstream: en amont

bank: berge