Académie de Toulouse - sections européennes - session 2016

Subject n°1

SEQUENCES

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

On the 1st of January, John subscribes to a new social network and accepts two friends in his circle.
On the 2nd of January, each of these friends offers him to make friends with two other people and he accepts them.
On the 3rd of January, each of these new friends (only those from the previous day) offers him, again, to make friends with two other people and he accepts them.

Let's assume that the progression goes on this way.

Answer the following questions . In all cases justify your answer.

1) What is the number of friends in John's circle on the 1st day, 2nd day and 3rd day of January ?

2) We want to model this situation by a sequence (u_n) .
Let u_n be the number of friends on the nth day.

Which of the following three expressions would fit for u_n ?

a) $\forall n \in \mathbb{N}, u_n = 2n$ b) $\forall n \in \mathbb{N}, u_n = 2^{n+1}$ c) $\forall n \in \mathbb{N}^*, u_n = \sum_{k=1}^{k=n} 2^k$

3)How many friends are there, in total, in John's circle on the 15th of January ?

4)Theoretically, when is it possible for John to be friends with all the people in France ?

Information : nowadays, the population of France is about 66 million.