

Subject 5

Please, don't write on the exam paper.

Let's consider the equation $(E): x^4 = e^x$, where $x \in \mathbf{R}$.

The aim of this exercise is to determine the number of solutions of equation (E) and to locate them.

1. Using your graphic calculator, conjecture the number of roots of equation (E) .
2. Show that $f : x \rightarrow e^x - x^4$ is strictly increasing over the interval $] -\infty; 0]$.
Calculate $f(0)$ and $\lim_{x \rightarrow -\infty} f(x)$.
How many roots does equation (E) have in the interval $] -\infty; 0]$?
3. Show that $x_0 > 0$ is a solution of equation (E) if and only if x_0 is a solution of the equation

$$\ln x - \frac{x}{4} = 0$$

Let g be the function $g : x \rightarrow \ln x - \frac{x}{4}$.

Fill in the table below:

| | | | | | |
|---------|---|---|---|---|---|
| x | 0 | | 4 | | $+\infty$ |
| $g'(x)$ | | | + | 0 | - |
| g | | | | ? | |
| | | ? | | | ? |

Show that the graph of g intersects the x -axis twice.

4. Conclusion

How many roots does equation (E) have in \mathbf{R} ?

With your calculator, give the approximate value of these roots to the nearest hundredth.