

**Subject n°7**

**Please, don't write on the exam paper.**

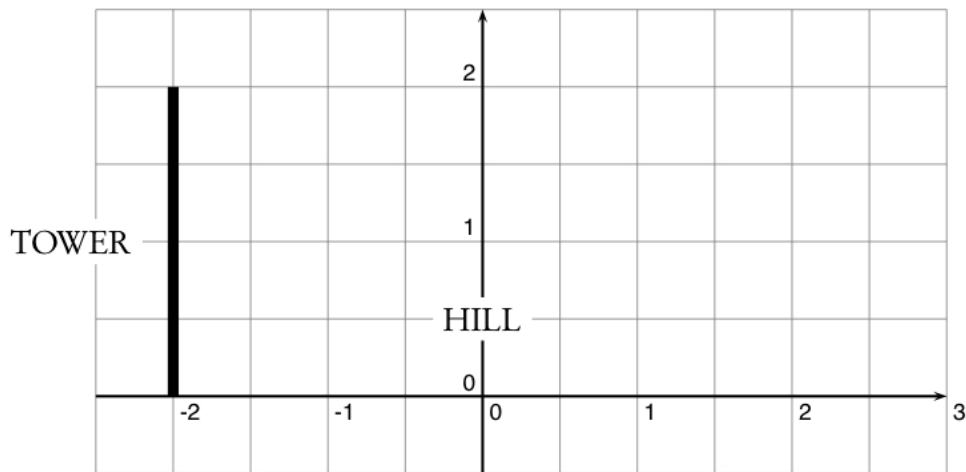
In the diagram below, the  $x$ -axis represents the flat ground.

There's a hill whose profile is given by  $y = 1 - x^2$  for  $-1 \leq x \leq 1$ .

There's a tower whose basis is at  $(-2 ; 0)$  and whose top is at  $(-2 ; 2)$ .

The aim of the exercise is to find how close to the origin on the positive  $x$ -axis we can come and still see (just barely) the top of the tower.

1. Complete the diagram below by drawing the profile of the hill.  
Place approximately the closest point to the origin on the positive  $x$ -axis, where we still can see (just barely) the top of the tower. Name this point  $G$ .



2. Consider the number  $a$  with  $0 < a < 1$  so that the line connecting the top of the tower at  $(-2; 2)$  and  $P(a; 1 - a^2)$  is tangent to the parabola that represents the hill's profile at point  $P$ .  
Show that  $a$  is one solution to the equation:  $a^2 + 4a - 1 = 0$ .  
Find  $a$ .
3. Show that the equation of the tangent to the parabola at point  $P$  can be written as :  
$$y = -2ax + 1 + a^2.$$
4. Find out the exact value of the abscissa of point  $G$ .