

Sujet N°22

Please do not write on the exam paper, and do not forget to give it back at the end of the test.

FUNCTIONS

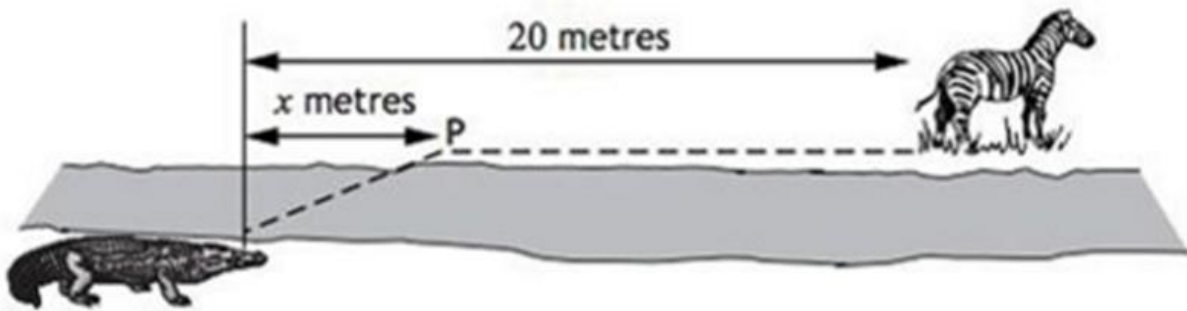
An African problem

A crocodile is stalking a prey located 20 metres further upstream on the opposite bank of a 6 metres wide river.

Crocodiles travel at different speeds on land and in river. The speed in river is 2 meters per second and the speed on land is 2.5 meters per second.

We recall the formula relating speed, distance and time: $v = \frac{d}{t}$.

The time taken by the crocodile to reach its prey can be minimized if it swims to a particular point P. This point P is x metres upstream on the other side of the river as shown in the diagram below.



In a first step, we admit that the time taken, T , measured in seconds, is given by:

$$T(x) = 0.5\sqrt{36 + x^2} + 0.4(20 - x)$$

1.a. Calculate the time taken if the crocodile swims only.

b. Calculate the time taken if the crocodile swims the shortest distance possible.

2. Between these two extremes there is one value of x which minimizes the time taken.

Using your calculator, try to find out an approximate value for x and hence the minimum possible time.

3. Prove the formula giving T with respect to x .

4. By studying the variations of the function T , prove the result of question 2.

Vocabulary:

to stalk: traquer

prey: proie

upstream: en amont

bank: berge