

Subject n°28: Functions

Please do not write on this document, and do not forget to hand it back at the end of the test.

A company has received a new machine which complexity requires a progressive learning process. This machine continuously produces objects and its production evolves with time. A study is made during the first five months. We note  $x$  the number of months since the installation of the machine. The function giving the number (in thousands) of objects produced by this machine is defined on

$[0 ; 5]$  by  $f(x) = \frac{100x}{x+1}$ .

1°) Prove that the derivative function  $f'$  of  $f$  on  $[0 ; 5]$  can be written as  $f'(x) = \frac{100}{(x+1)^2}$ .

2°) Study the sign of  $f'(x)$  on  $[0 ; 5]$  and deduce the table of variations of  $f$ .

3°) Copy and complete the following table of values (*you will round the results to the nearest unit*).

$x$	0	1	2	3	4	5
$f(x)$						

4°) Represent graphically the function  $f$ . You will take for units 2 cm per month on the x-axis and 1 cm for 10,000 objects on the y-axis.

5°) We estimate that the machine is profitable if it produces at least 80,000 objects. Determine the first month when the machine is profitable.