Académie de Toulouse – sections européennes – session 2017

Subject n°1

Please do not write on this document, and do not forget to hand it back to the jury at the end.

PROBABILITIES

In 1936, in order to predict the result of the presidential election, the American magazine *Literary Digest* organized a "Straw vote", by sending by mail approximately ten million ballot papers. The sample of this poll relied on different sources: a list of subscribers to the magazine, phone book, car registration.

Out of the ten million poll papers, the magazine received around 2.4 million answers. Only 41 % of the people who answered indicated that they would vote for the past president, Franklin Roosevelt.

The magazine announced the fall of the president in function, but he was re-elected with more than 61 % of the votes!

We consider one person at random among the ten million who received the poll papers. We note S the event "The person participates to the poll" and R the event "The person votes for Roosevelt". We suppose that $P(R) = 0.61$.

1º) Draw a probability tree and deduce from the text $P(S)$ and $P_S(R)$.

2º) Calculate $P(R \cap S)$. Interpret this result.

3º) Deduce from it, rounding to the second decimal, that $P(\bar{S} \cap R) = 0.51$.

4º) Calculate $P_{\bar{S}}(R)$. Interpret this result and compare it to $P_S(R)$.

5º) Calculate $P_R(S)$ and $P_{\bar{R}}(S)$. Compare and interpret these two results.

Information : despite the gigantic size of the sample used by the magazine *Literary Digest*, it presented a few flaws and predicted the wrong result. At the same time, the Gallup institute made a poll based on a "representative" sample of a few thousand people and predicted Roosevelt's victory. It was the beginning of the use of scientific methods for polls.

<u>Vocabulary:</u>

ballot papers: bulletins de vote

flaws: failles

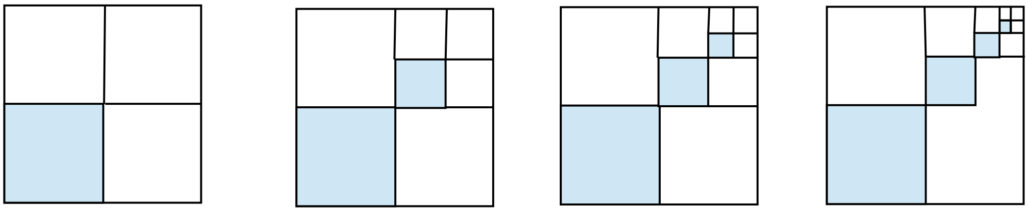
Académie de Toulouse – sections européennes – session 2017

Sujet n°2

Please do not write on this document, and do not forget to hand it back to the jury at the end.

SEQUENCES

In the first step, we colour one quarter of a white square of side 1 m. We continue, step by step, by colouring another square of area one quarter of the previous coloured one :



For each integer $n \geqslant 1$, we note a_n the area of the square which has **just been coloured** at step *n*, and s_n the **total coloured** area at step *n*.

1°) What is the variation of the sequence (a_n) ? And that of the sequence (s_n) ?

2°) **a)** Determine a_1 ; a_2 ; a_3 and a_4 . (Give the results as fractions).

b) What is the nature of the sequence (a_n) ? Precise its first term and its ratio.

c) Deduce from this a_n in terms of *n*.

3°) **a)** Determine s_1 ; s_2 ; s_3 ; s_4 . (Give the results as fractions).

b) Determine the expression of s_n in terms of *n*.

4°) **a)** What is the limit of the sequence (s_n) ?

b) What is the first step when at least 33/100 of the square is coloured?

Tip:

- If (u_n) is an arithmetic sequence, $S_n = u_1 + u_2 + \cdots + u_n = n(u_1 + u_n)/2$;
- If (u_n) is a geometric sequence, $S_n = u_1 + u_2 + \cdots + u_n = u_1 \times (1 - q^n)/(1 - q)$.

b) What will be coloured.

Académie de Toulouse – sections européennes – session 2017

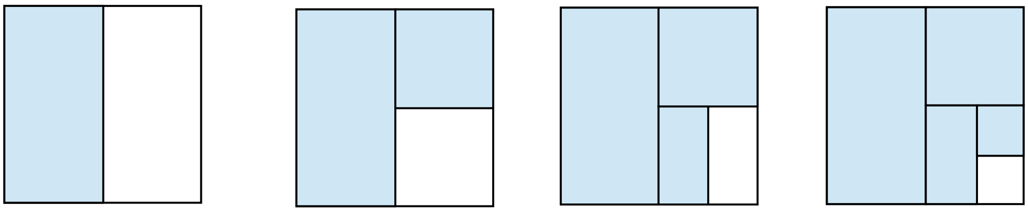
Sujet n°3

Please do not write on this document, and do not forget to hand it back to the jury at the end.

SEQUENCES

In the first step, we colour one half of a white square of side 1 m. We continue, step by step, by colouring one half of the remaining uncoloured surface :

Step 1 Step 2 Step 3 Step 4



For each integer $n \geqslant 1$, we note a_n the area of the surface which has **just been coloured** at step *n*, and s_n the **total coloured** area at step *n*.

1°) What is the variation of the sequence (a_n) ? And that of the sequence (s_n) ?

2°) **a)** Determine a_1 ; a_2 ; a_3 and a_4 . (Give the results as fractions).

b) What is the nature of the sequence (a_n) ? Precise its first term and its ratio.

c) Deduce from this a_n in terms of *n*.

3°) **a)** Determine s_1 ; s_2 ; s_3 ; s_4 . (Give the results as fractions).

b) Determine the expression of s_n in terms of *n*.

4°) **a)** What is the limit of the sequence (s_n) ?

b) What is the first step when at least 99% of the square is coloured?

Tip:

- If (u_n) is an arithmetic sequence, $S_n = u_1 + u_2 + \cdots + u_n = n(u_1 + u_n)/2$;
- If (u_n) is a geometric sequence, $S_n = u_1 + u_2 + \cdots + u_n = u_1 \times (1 - q^n)/(1 - q)$.

Académie de Toulouse – sections européennes – session 2017

Subject n°4

Please do not write on this document, and do not forget to hand it back to the jury at the end.

FUNCTIONS

Aphids attack.

A lot of aphids are invading a rose garden.

Some ladybugs, predators of aphids, are introduced in this rose garden. Some studies have shown the following result:

if t denotes the number of days since the introduction of the ladybugs, then the number of aphids (in thousands) is given by the function f, such that $f(t)=0.003t^3-0.12t^2+1.1t+2.1$.

The speed of proliferation of aphids at the instant t is given by $f'(t)$, (in thousands of aphids per day).

1) What is the number of aphids when the ladybugs are introduced in the rose garden?

2) Express the speed of proliferation in terms of t.

Study its sign over the interval [0 ; 20]. Give an interpretation.

3) Study the variations of f over [0 ; 20].

When will the number of aphids start to decrease?

4) We estimate that aphids are not a problem anymore when their number is inferior to 1,000. Determine when the number of aphids becomes inferior to 1,000.

<u>Vocabulary:</u>

aphids: pucerons

ladybugs: coccinelles

Sujet n°6

Please do not write on the subject paper and do not forget to give it back at the end.

Statistics

The table below gives information about the average life expectancy (in years) in Germany, Iraq and the United Kingdom.

	1940	1950	1960	1970	1980	1990	2000	2010	2015
Germany	62	67	69	70	73	75	78	80	81
Iraq	31	37	48	57	55	67	70	68	69
UK	60	68	71	72	74	75	78	80	81

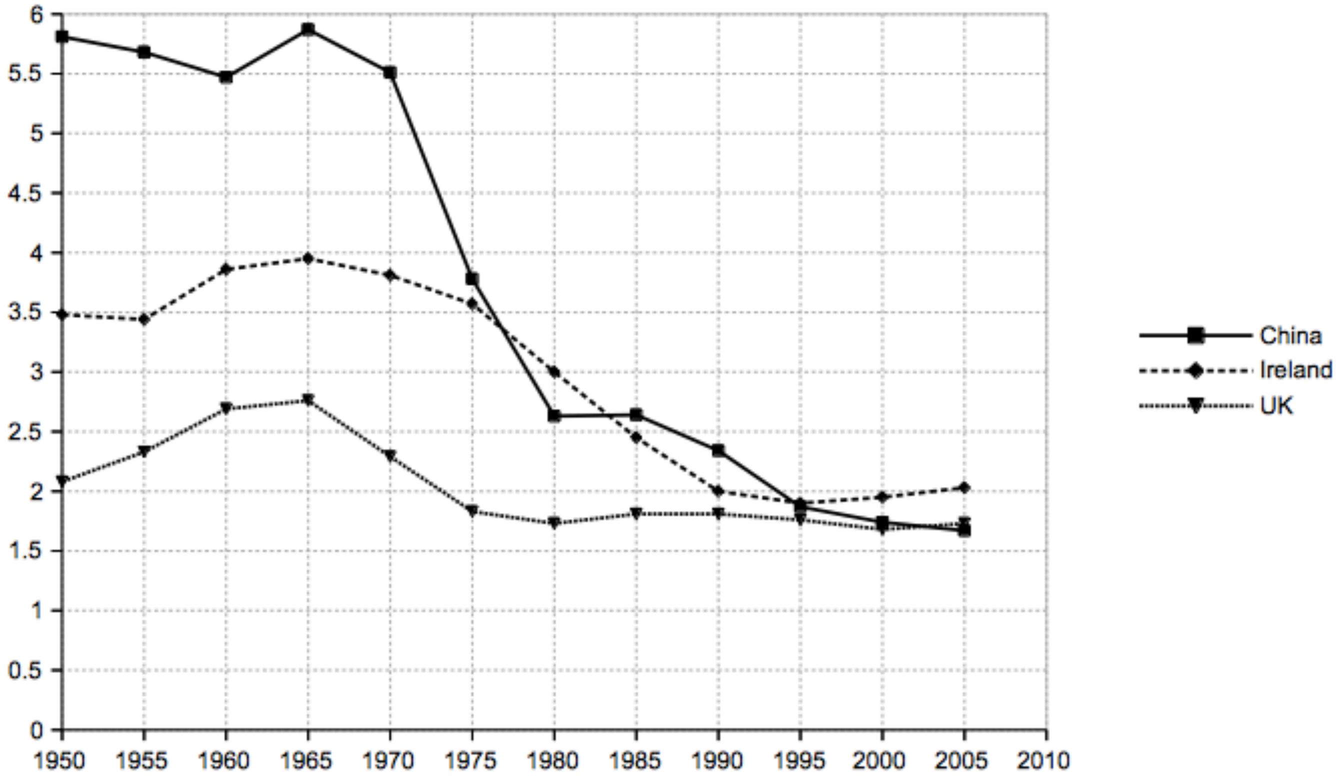
From https://en.wikipedia.org

1. Compute the global evolution rate between 1940 and 2015 for the three countries. Compare.
2. For each country, determine :
   a. The median over these 9 years
   b. The lower and upper quartiles
   c. Compare the data per country.

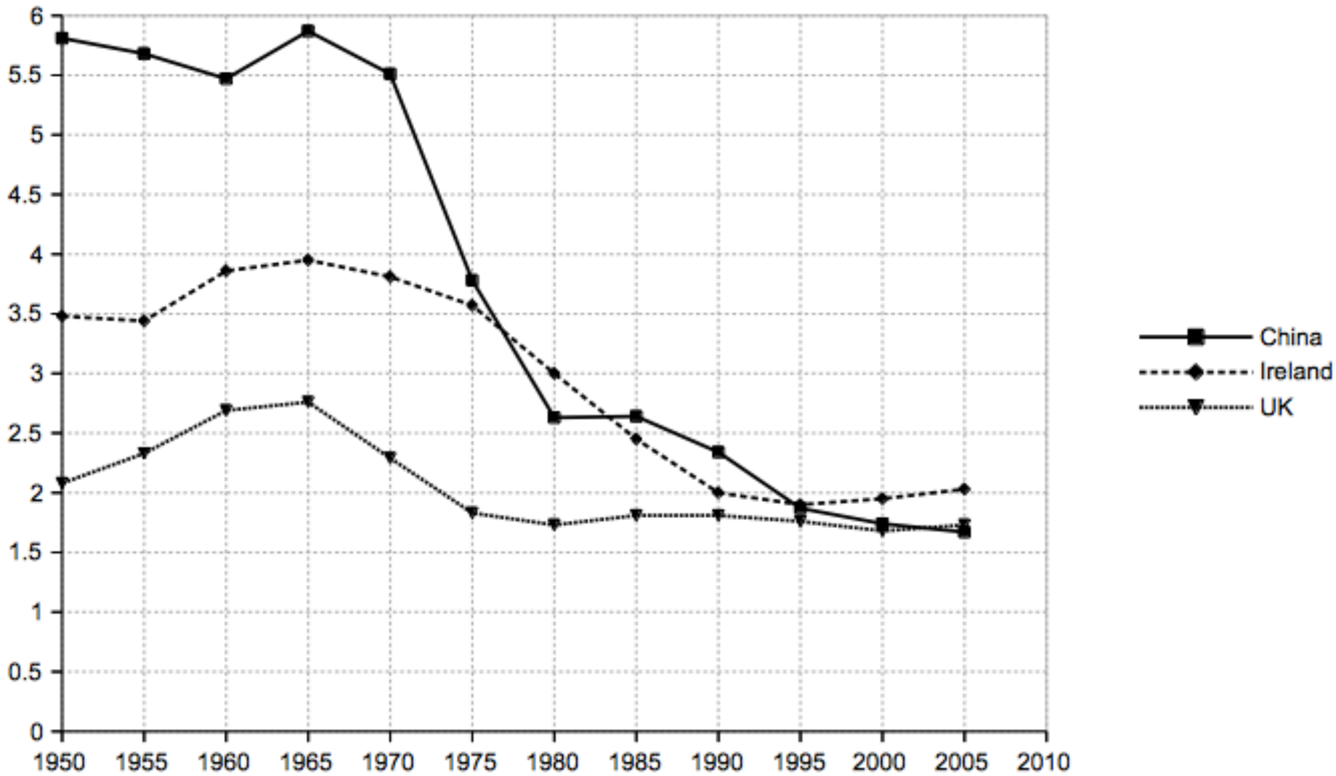
The data below gives information about the average number of children born per woman.

	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010
China	5.81	5.68	5.47	5.87	5.51	3.78	2.63	2.64	2.34	1.87	1.74	1.67	1.6
Ireland	3.48	3.44	3.86	3.95	3.81	3.57	3	2.45	2	1.9	1.95	2.03	2.11
UK	2.08	2.33	2.69	2.76	2.29	1.83	1.73	1.81	1.81	1.76	1.68	1.73	1.86

1. Compute the global evolution rate between 1940 and 2010 for the three countries. Compare.
2. These data are now shown in the graph given on an attached paper. Plot the average number of children born per woman in 2010. Comment on this graph.



6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
China
Ireland
UK



6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
China
Ireland
UK

Sujet n°7

Please do not write on the subject paper and do not forget to give it back at the end of the test.

Functions

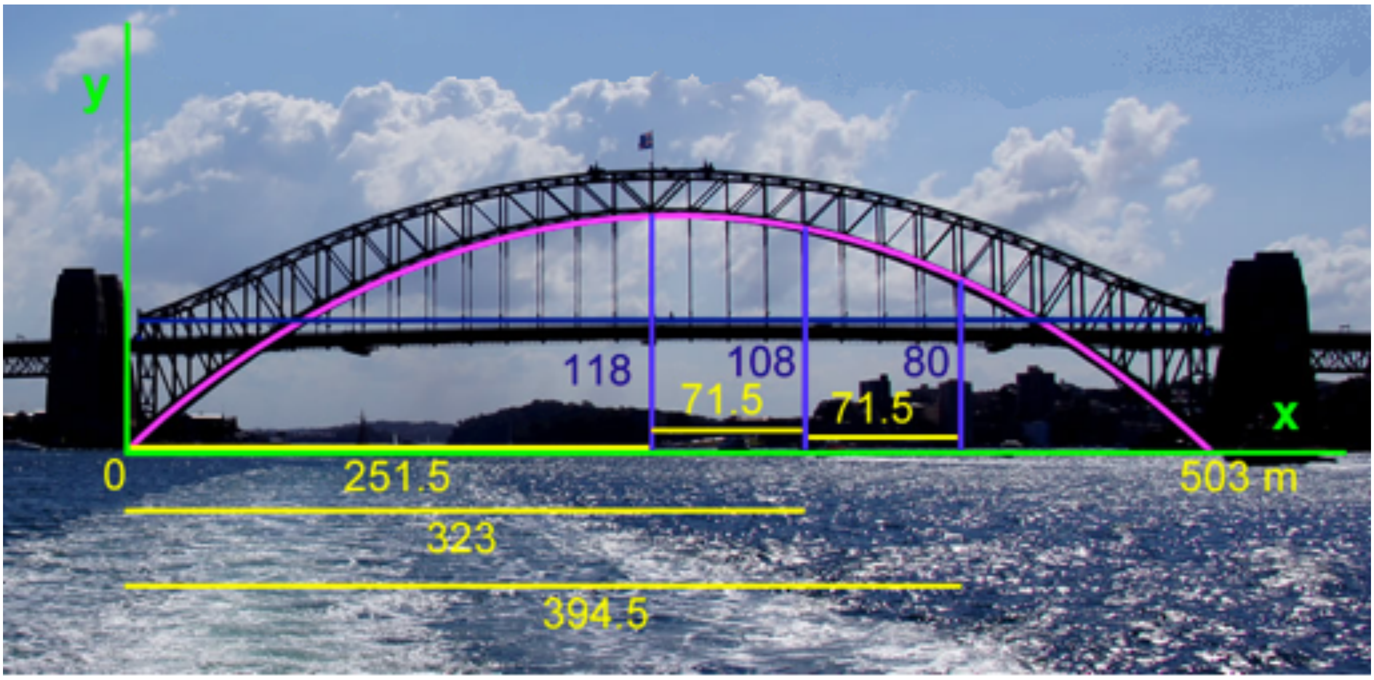


image from passyworlofmathematics.com

On this picture, we can see Sydney Harbour bridge (near the Opera).

White numbers are horizontal data and black numbers are vertical data.

Knowing that the under arch of this bridge is a parabola, find its equation.

Sujet n°8

Please do not write on the subject paper and do not forget to give it back at the end of the test.

Probabilities

In a report dated from 1999, July 8th, the European Commission studied the evaluation of a test to diagnose BSE (Bovine spongiform encephalopathy) which causes a lot of deaths in cattles.

- 70% of infected cerebral animal samples are positive to the W test.
- 90% of non infected cerebral animal samples are negative to the W test.
- In a cattle, 7% of cows are infected.

Let's assume that one animal is randomly tested in this cattle.

Let's label I " the chosen animal is infected " and $\bar{I}$ its contrary.
Let's label T " the test is positive " and $\bar{T}$ its contrary.

Each probability has to be rounded to the thousandth if needed.

1. Draw a probability tree of this situation.
2. Compute the probability that a cow is sick and its test is positive.
3. Compute the probability that the test is positive.
4. Knowing that the test is positive, what is the probability that the cow is sick ?

Vocabulary :
A cattle = un troupeau

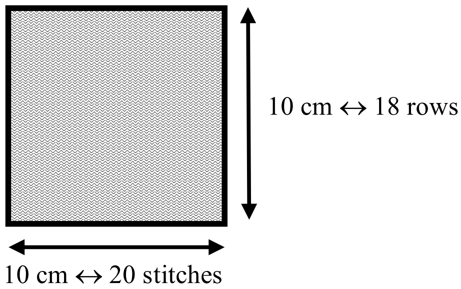
Académie de Toulouse – sections européennes – session 2017

Sujet n°9

Please do not write on the subject paper and do not forget to give it back at the end of the test.

Sequences

Lisa loves knitting. She wants to give a knitted scarf to her grand-mother for Christmas. So she plans her job…
The scarf has to be 40 cm wide and 1.2 meter long.
With the wool she chooses, Lisa can knit a squared sample of 10 cm side. This sample is shown just below.



The first hour, Lisa knits 50 stitches. And each next hour, she is better with her needles so she can knit 10 stitches more than the previous hour.

Let's label u_n the number of stitches knitted the n-th hour. So $u_1 = 50$.

1) Compute how many stitches are needed for the width of the scarf and how many rows for its length.
2) How many stitches are hence needed for the whole scarf ?
3) Compute how many stitches Lisa will knit the second hour, and then the third hour.
4) Model
   a) What kind of progression is u_n ?
   b) Determine the recursive formula linking u_{n+1} and u_n.
   c) Find the formula linking u_n and n.
5) Determine the number of hours Lisa will need to knit her grand-mother's gift.

Vocabulary :
To knit = tricoter
A stitch = une maille
A needle = une aiguille

Tip :

- if (u_n) is an arithmetic sequence, $S_n = u_1+u_2+ \ldots+u_n = n(u_1+u_n)/2$
- if (u_n) is a geometric sequence, $S_n = u_1+u_2+ \ldots+u_n = u_1*(1 - q^n)/(1 - q)$

Sujet n°10

Please do not write on the subject paper and do not forget to give it back at the end of the test.

Functions

Alma sells apple juice. It costs her 20 pennies to make each cup. If she charges 60 pennies per cup, then 70 people will buy a cup. If she charges £2.00 per cup, then only 35 people will buy a cup.

Let's assume that the number of people who buy apple juice is a linear function of the price. That means $N(p) = a\,p + b$ where N is the number of people buying apple juice and p is the selling price in pounds.

1. Solve the simultaneous equations to find the N function.
2. The profit is given by the expression $B(p) = N(p)*(p - 0.2)$ Explain this formula.
3. How much should Alma charge to maximize her profit ? How many people will buy her apple juice ? What will her profit be ?

Académie de Toulouse – sections européennes – session 2017
Sujet n° 11

Please do not write on the subject, and do not forget to hand it back at the end.

Sequences

In the town of Newcity the number of households was 10,000 at the end of 2010,

10,300 at the end of 2011,

and 10,609 at the end of 2012.

1) Supposing that the population keeps on following this progression, what kind of sequence can we use to model the number of households?

In the same time: 303 new houses were built during the year 2011,

311 during the year 2012,

and 319 during the year 2013.

2) Supposing that the construction of new houses keeps on following this progression, what kind of sequence can we use to model it?

3) Give the number of households at the end of 2015, and the number of new houses built by the end of 2015 (round it to the nearest unit if necessary).

4) Do the same for 2020.

5) We know that in 2010 there was no shortage of housing.

Study the evolution of the situation.

vocabulary:

shortage: pénurie

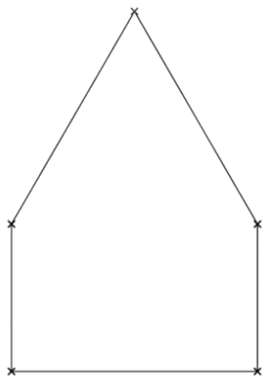
Académie de Toulouse – sections européennes – session 2017
Sujet n° 12

Please do not write on the subject, and do not forget to hand it back at the end.

Functions

I want to build a new window in my bedroom.

Since the sidewall of my bedroom looks like an equilateral triangle, I'd like my new window to be made of a rectangle with an equilateral triangle just above it. Here is a sketch of this shape:



I would like to have the biggest window possible, but I only have 10 m of frame to build my window with.

1) Choose notations to label the dimensions of the window.

2) Use the constraint given on the perimeter of the window to express one of the dimensions in terms of the other.

3) Find a function which gives the area in terms of one of the dimensions.

4) Find the dimensions which maximise the area of the window. (round to the nearest cm).

<u>vocabulary:</u>

frame: précadre, encadrement

Académie de Toulouse – sections européennes – session 2017
Sujet n° 13

Please do not write on the subject, and do not forget to hand it back at the end.

Probabilities

In my street the bins are collected in the morning and we are supposed to put them out just the evening before.
But it is very windy, so I know that if I put my bin out in the evening I get three chances out of five that it will be tipped over by the wind and then will not be collected.
So three times out of ten I put my bin out in the morning.
But when I put the bin out in the morning, half the time I am late and the bin is eventually not collected.

1) Propose a model to study the situation of one collection, with events and probabilities.
Then draw a probability tree.
2-a) I put my bin in the evening, what is the probability that it will be collected the next morning?
b) What is the probability that my bin shall be collected on any collection day?
c) My neighbour sees that my bin hasn't been collected on a collection day, what is the probability that I have put it out the evening before? (round to the nearest hundredth).

Additional question:
In my street there are twenty houses, and all the inhabitants have the same habits and the same trouble with their bins, even though we all act independently from the others.
Let X be the random variable counting the number of bins collected on a collection day.
What is the probability law of X?
How many bins shall be collected on average any collection day?
What do you think of that result?

Académie de Toulouse – sections européennes – session 2017
Sujet n° 14

Please do not write on the subject, and do not forget to hand it back at the end.

Reading and Calculating

Three friends, Alicia, Ben and Chris, want to travel from Toulouse to London.

Alicia decides to ride her motorcycle from Toulouse city hall to Matabiau railway station, then she will go to Paris by train (TGV). She will arrive in Montparnasse railway station. From there she'll go to Paris-Nord station on the Underground, and travel with the Eurostar to St Pancras station in London. Then she'll use the tube to go to the British Museum.

Ben decides to drive from Toulouse city hall to Blagnac airport. Then he will fly to Heathrow airport, and take a cab to go to the British Museum.

Chris decides to drive from Toulouse city hall to Calais, and cross the channel on a ferryboat. Then he will drive from Dover to London. He will drive round to Regent's park, park there and go down by foot to the British Museum.

Who makes the most environmentally friendly choice?

<u>To help you explain your answer you are given the following data:</u>

Toulouse City Hall to Matabiau station:

1.5 km.

Matabiau station to Montparnasse station: 713 km.

Montparnasse station to Paris-Nord station: 5.3 km.

Paris-Nord station to the Channel Tunnel: 333 km.

Channel Tunnel: 88km.

Channel Tunnel to St Pancras station:

68 miles.

St Pancras station to the British Museum: 0.8 miles.

Toulouse city hall to Blagnac airport: 9 km.

Blagnac airport to Heathrow airport: 886.17 km

Heathrow airport to the British Museum: 17.1 miles.

Toulouse city hall to Calais: 965 km.

Calais to Dover: 30.1 miles.

Dover to Regent's park: 80.1 miles.

1 km = 0.621371 mile and 1 mile = 1.60934 km.

CO_2 emissions for the average Eurostar train: 11.2 g per passenger per km.

CO_2 emissions for the average TGV train: 3.2 g per passenger per km.

CO_2 emissions for the average subway train: 3.8 g per passenger per km.

CO_2 emissions for the average short-haul international flight: 26864 g per passenger per km.

CO_2 emissions for a ferryboat for one average car: 750 g per mile.

grams of CO_2 emissions for Alicia’s motorcycle per km (driver alone): 100

grams of CO_2 emissions for Ben’s taxi per km (one passenger only): 222

grams of CO_2 emissions for Chris’ and Ben’s car per km (driver alone): 169.388

Académie de Toulouse – sections européennes – session 2017
Sujet n° 15

Please do not write on the subject, and do not forget to hand it back at the end.

Statistics

On a trip to London two friends decide to use the ten most popular stations.
They manage to find the following data:

Estimates of Station Usage 2015-16

OFFICE OF RAIL AND ROAD

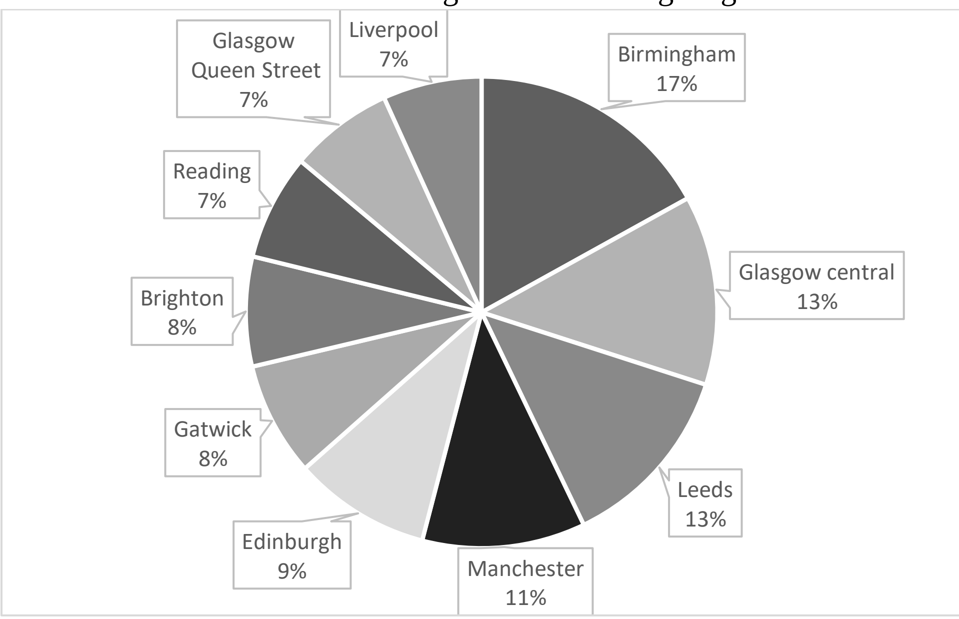
Publication date: 6 December 2016

www.orr.gov.uk/statistics/published-stats/station-usage-estimates

Stations with the most entries & exits

Within London		Outside London	
Station	Entries & Exits	Station	Entries & Exits
1. Waterloo	99,148,388	1. Birmingham New Street	39,077,018
2. Victoria	81,151,418	2. Glasgow Central	30,000,582
3. Liverpool Street	66,556,690	3. Leeds	29,723,734
4. London Bridge	53,850,938	4. Manchester Piccadilly	25,792,700
5. Euston	41,677,870	5. Edinburgh	21,723,960
6. Stratford	41,113,260	6. Gatwick Airport	18,028,846
7. Paddington	36,536,074	7. Brighton	17,333,326
8. King's Cross	33,361,696	8. Reading	16,755,984
9. Clapham Junction	32,282,220	9. Glasgow Queen Street	16,424,064
10. St. Pancras	31,723,686	10. Liverpool Central	15,638,894

1) What kind of data is it?
2) Draw a pie chart. Use the circle you have been given by the jury.
3) One of the friends draws a pie chart to visualise the other data: the ten stations with the most entries & exits outside London. She gets the following diagram:



She claims that the two diagrams look strangely the same.

a) Do you agree with her? (explain your answer)

Then she wants to see if the similarity holds when she looks at all the stations, not just the most popular.

She gets the following numbers:

total entries & exits per London stations : 1,471,957,408

total entries & exits per outside London stations : 1,455,597,014

b) Calculate the part represented by the total of the ten most popular London stations among all London stations. (round to the nearest percent)

c) Calculate the part represented by the total of the ten most popular stations outside London among all the stations outside London. (round to the nearest percent)

d) Compare your results.

Additional question:

The other person wants to try and explain why some stations outside London are the most popular. She claims that they are the most popular because they are in a highly populated area.

To make her point she gets the population for the great built-up areas around each station (census of 2011):

around Birmingham 2,440,986

around Glasgow 1,209,143

around Leeds 1,777,934

around Liverpool 864,122

around Manchester 2,553,379

around Brighton 474,485

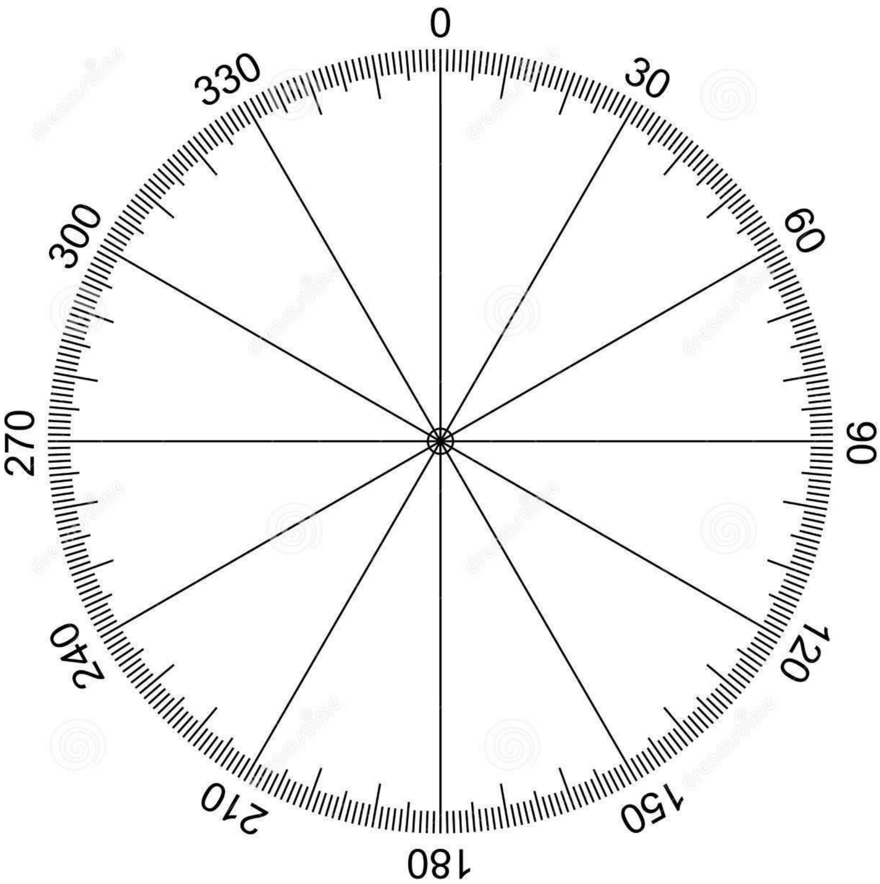
around Reading 318,018

She doesn't take Gatwick into account, because it is actually the place of a London airport.

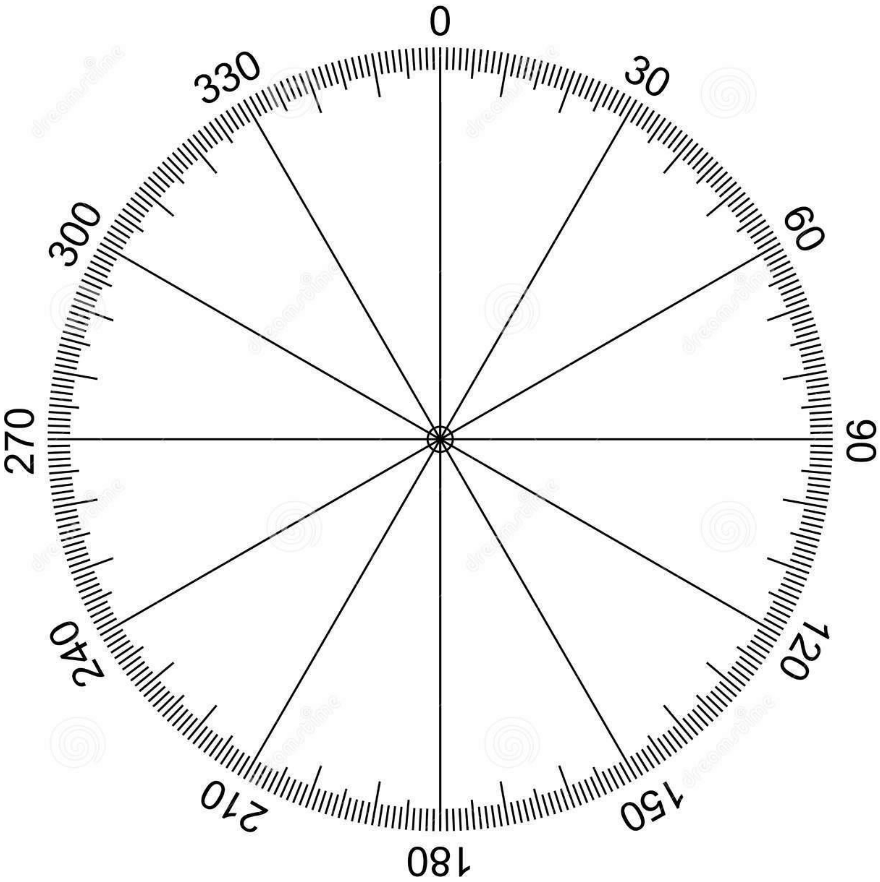
Do you think the data supports her claim? (explain your answer).

<u>vocabulary:</u>

census: recensement.



(image source:
dreamstime.com)



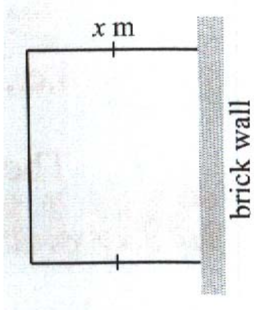
(image source:
dreamstime.com)

Sujet n°16

Please do not write on this document, and do not forget to hand it back to the jury at the end.

OPTIMIZATION

A vegetable gardener has 40 m of fencing to enclose a rectangular garden where one side is an existing brick wall. If the width is x m as shown :



1. Show that the enclosed area is given by $A(x) = -2x^2 + 40x$ m².
2. Find the width x so that the vegetable garden has a maximum area.
3. What is the garden's maximum area?

This honorable gardener also possesses a greenhouse to grow flowers. The temperature T° Celsius in the greenhouse t hours after dusk (7.00 pm) is given by :

$$T(t) = \frac{1}{4}t^2 - 5t + 30 \quad (\text{with } t \leq 20)$$

4. What is the temperature in the greenhouse at dusk?
5. At what time is the temperature minimal?
6. What is the minimum temperature?

Fencing : clôture ;
A greenhouse : une serre ;
At dusk : au crépuscule / à la tombée de la nuit.

Sujet n°17

Please do not write on this document, and do not forget to hand it back to the jury at the end.

SEQUENCES

Harvey starts a new job selling TV sets. He hopes to sell 11 sets in the first week, 14 the next one, 17 the next one, etc. (in a regular pattern).

1. What is the nature of the sequence $u(n)$ representing TV sets sold in a given week n (with $n = 1, 2, 3, \ldots$)?
2. What are the characteristics of this sequence?
3. What is the general term of this sequence?
4. Show that the total number of TV sets sold between the first week and a given week n is given by:
$$S = \frac{n(22 + 3(n-1))}{2}$$
5. In what week will Harvey hope to sell his 2000th TV set?

Tip:

- If (u_n) is an arithmetic sequence, $S_n = u_1 + u_2 + \cdots + u_n = n(u_1 + u_n)/2$;
- If (u_n) is a geometric sequence, $S_n = u_1 + u_2 + \cdots + u_n = u_1 \times (1 - q^n)/(1 - q)$.

Académie de Toulouse – sections européennes – session 2017

Sujet n°23

Please do not write on this document, and do not forget to hand it back to the jury at the end.

FUNCTIONS

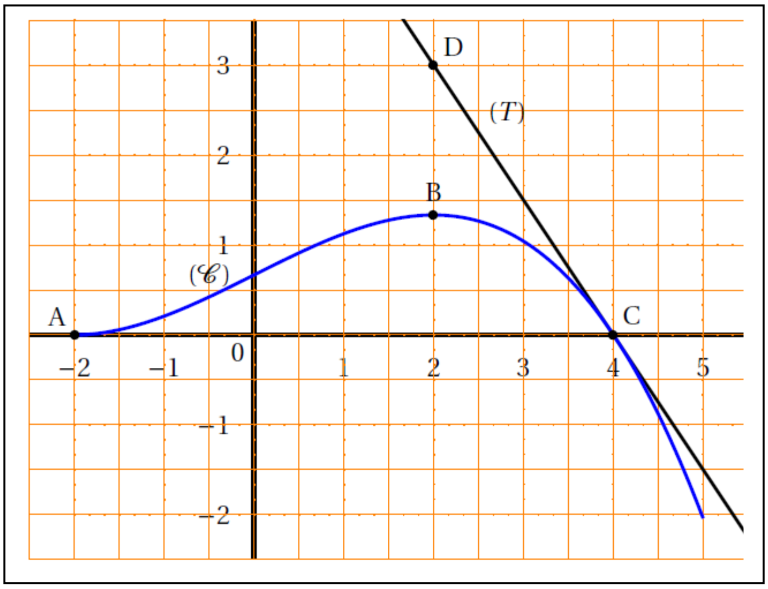
This is a true or false exercise. For each of the 4 statements, you need to say if it is either true or false and justify your answer.

We consider a function *f* defined and differentiable on the interval $[-2; 5]$, increasing on $[-2; 2]$ and decreasing on $[2; 5]$.

We denote *f* ' the first derivative of *f*.

The graph ($\mathscr{C}$) of *f* in an orthonormal set is given below. It passes through points $A(-2; 0)$, $B\left(2; \frac{4}{3}\right)$ and $C(4; 0)$.

($\mathscr{C}$) admits at points *A* and *B* a tangent that is parallel to the *x*-axis and tangent (*T*) at point *C* passes through point $D(2; 3)$.



Statement 1: $f'(4) = -\frac{2}{3}$

Statement 2: $f \text{ is a concave function on } [-2; 2]$

Statement 3: $2 \leq \int_1^3 f(x)dx \leq 3$

Statement 4: $\text{The equation } f(x) = \ln 2 \text{ does not have any solution on } [-2; 5]$

Sujet n°24

Please do not write on this document, and do not forget to hand it back to the jury at the end.

LOGIC - PROBABILITY

THE BOY OR GIRL PARADOX

Imagine that a family has two children, one of whom we know to be a boy. What then is the probability that the other child is a boy? The obvious answer is to say that the probability is 1/2—after all, the other child can only be *either* a boy *or* a girl, and the chances of a baby being born a boy or a girl are essentially equal. In a two-child family, however, there are actually four possible combinations of children: two boys (MM), two girls (FF), an older boy and a younger girl (MF), and an older girl and a younger boy (FM). We already know that one of the children is a boy, meaning we can eliminate the combination FF, but that leaves us with three equally possible combinations of children in which *at least* one is a boy—namely MM, MF, and FM. This means that the probability that the other child *is* a boy—MM—must be 1/3, not 1/2.

Do you agree or not with this explanation?

Develop your point of view

Académie de Toulouse – sections européennes – session 2017

Sujet n°25

Please do not write on this document, and do not forget to hand it back to the jury at the end.

PROBABILITY

A particular brand of spread[1] is produced in three varieties: standard, light and very light.

During a marketing campaign, the manufacturer advertises that some cartons of spread contain coupons worth £1, £2 or £4.

For each variety of spread, the proportion of cartons containing coupons of each value is shown in the table.

	Variety		
	Standard	Light	Very light
No coupon	0.70	0.65	0.55
£1 coupon	0.20	0.25	0.30
£2 coupon	0.08	0.06	0.10
£4 coupon	0.02	0.04	0.05

For example, the probability that a carton of standard spread contains a coupon worth £2 is 0.08.

In a large batch of cartons, 55 per cent contain standard spread, 30 per cent contain light spread and 15 per cent contain very light spread.

A carton of spread is selected at random from the batch. Find the probability that the carton:

1. Contains standard spread and a coupon worth £1;
2. Does not contain a coupon;
3. Contains light spread, given that it does not contain a coupon;
4. Contains very light spread, given that it contains a coupon.

[1] Spread: pâte à tartiner

Sujet n°27

Please do not write on this document, and do not forget to hand it back to the jury at the end.

FUNCTIONS

A company that produces sails for leisure sailing boats wishes to make a two-colour model where the surface of each colour is identical.

Let *f* be the function defined on [0; 1] by: $f(x) = 2 - 2x$.

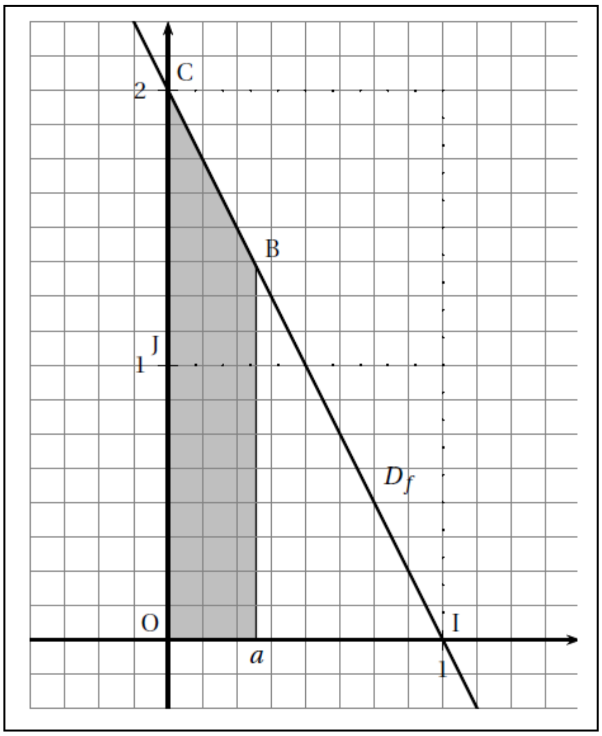
We have drawn below the straight line D_f, which is a graphic representation of function *f* in an orthonormal set (O, I, J). The coordinates of point *C* are $(0; 2)$.

Δ is the part of the plane which is inside triangle *OIC*.

Let *a* be a real number between 0 and 1. We mark *A* the point with coordinates $(a; 0)$ and *B* the point of D_f with coordinates $(a; f(a))$.

The aim of this exercise is to find the value of a, whereby segment [*AB*] divides Δ in two parts of equal surface.

Determine the exact value of a, then the approximate value rounded to the nearest hundredth.



Académie de Toulouse - sections européennes - session 2017

Subject 28

Sequences and probability

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

An endurance competition consists in climbing stairs and picking up, at the same time, tokens placed on the steps.
On the first step, there is one token, then on every following step, there is one more token than on the previous one.

a) You are the happy winner, after having climbed 100 steps and collected the whole amount of tokens in a record time of seven minutes and forty nine seconds .
Each collected token is worth £1, what is your gain ?

b) The second winner earns £3,828 , how many steps did he achieve to climb ?

c)The third winner doesn't earn any money, he has to toss a fair coin three times. If he gets at least two heads out of three trials, then he wins a computer, else he wins a Zorro costume. Is it more likely that the prize is a computer or a Zorro costume ?

Vocabulary :

token : jeton.

Subject 29

Sequences and probability

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

A charity fair is organised, people in charge of the raffles find it difficult to compute how much a ticket must be sold.

Prizes and their purchase price are as follows :

Prize	Purchase price per unit	Number of units
computer	£ 550	3
bicycle	£ 250	5
book	£ 25	19
bag of candies	£ 2	55

1) They expect to send 1,500 tickets and they want a total benefit of £ 4,000 for the lottery.

The printing cost for the whole pack of tickets is £15 .

What should be the price of each ticket ?

2) The charity fair began and James bought a ticket. The tickets will be drawn at five o'clock when all the tickets are sold. What is the probability for James to win a prize ?

Give the result to 3 decimal places.

3) What is the probability for Lola to win exactly one bicycle given that she bought 2 tickets ?

4) While his parents are trying their luck in the lottery, little Kevin is in the children's corner, throwing balls to make cans fall. The game becomes harder and harder, that is the reason why the reward for the first step is 1 lollipop, for the second step 2 more lollipops, for the third one 4 more lollipops , and so on...each time doubling .

In total, he got 31 lollipops, how many steps did he achieve ?

Vocabulary :

- raffles : tombola
- purchase price : prix d'achat

Académie de Toulouse - sections européennes - session 2017

Subject 30

Functions

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

A company produces 3D televisors.
The cost of production, $C(n)$, presented in thousands of pounds for n items, is given as follows by the function C :

$$C(n)=0.02n^2-2n+98 \quad \text{with} \quad n\in[50\,;150] \ .$$

1. Let us denote $S(n)$ the selling price, presented in thousands of pounds for n items.
Each item being sold £ 1,500, calculate $S(n)$.

2. The earnings, for n sold items are denoted $B(n)$.
Give the expression of $B(n)$ in terms of n.

3. Determine the values of n for which the production is profitable.

Subject 31

Functions

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

An artist built a fountain in which water describes a parabola, springing from the mouth of a fish, then passing through the hoop hold by a nymph at the highest point of the parabola, then falling down on a spotlight.

1.The center of the hoop is two metres above the water surface and the light projector is situated two metres away from the mouth of the fish. The fish and the projector are at water level.
Make a simple sketch of the situation above.

2.What is the equation of the parabola described by the water jet ?

3.The functions f_k are defined by :

$f_k(x)=(k+2)(-x^2+2x)$, $x \in \mathbb{R}$, *k* is a real number different from -2.

a)Show that the variations of f_k for $x \in [0;2]$ are independent from *k*.

b)Determine $f_0(x)$.

c)The diameter of the hoop is 40 cm. If the f_k functions represent the possible trajectories for the water jet, given that water must pass through the hoop, what are the possible values of *k* ?

Vocabulary :

hoop : large ring used as a toy or for circus performers to jump through.

Académie de Toulouse - sections européennes - session 2017

Subject 32

Probability, Sequences, Functions

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

Part A : Decide if each following statement is true or false, and justify your answer.

1. On Monday and Wednesday night, Mr. García went to bed listening to his favorite music CD. The CD contains music from 3 different composers: 3 tracks by Tchaikovsky, 7 tracks by Barber and 2 tracks by Gershwin. His stereo is programmed to play the tracks in a random order.

The likelihood that on both nights Mr. García fell asleep listening to the same composer is $\frac{31}{72}$.

2. Two fair dice are rolled.

Let A be the event : « obtaining a six with the first die » ,

and B the event : « obtaining an even number with the second die ».

The events A and B are not independent.

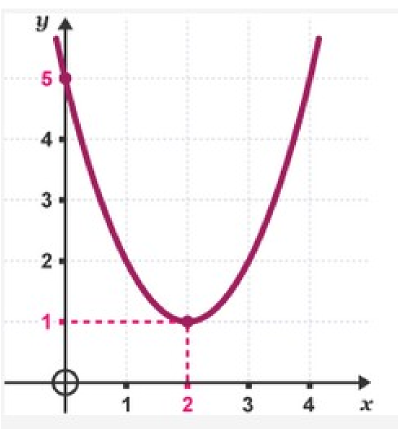
Part B :

1. Find the right answer :

The n^{th} *term of the sequence 0, 1, 4, 9....... is* :

a) $n^2 – 1$ b) $n^2 – n$ c) $(n^2 - 2n) + 1$ d) $(n^2 - 3n) + 2$

2. Find the equation of the parabola below :



Subject 35

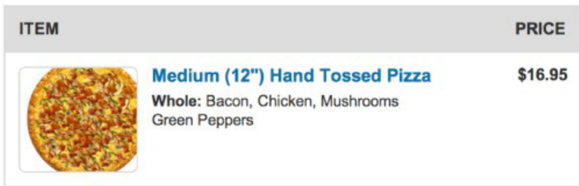
FUNCTIONS

Please do not write on this exam paper and give it back at the end of the test

PIZZAS

1. Below are the prices for a medium 2-toppings pizza and a medium 4-toppings pizza. How much do you think the restaurant is charging for one topping, and how much would you expect to pay for a plain pizza (i.e. with zero topping)?

ITEM	PRICE
Medium (12") Hand Tossed Pizza **Whole:** Pepperoni, Green Peppers	$13.97

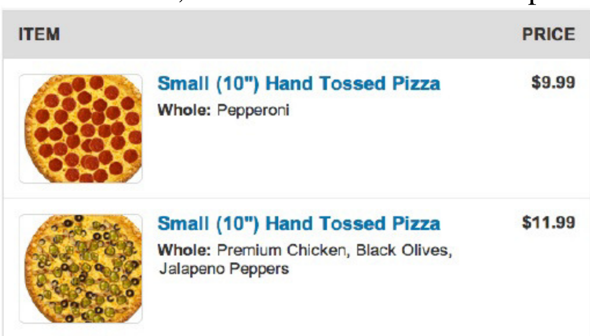


ITEM	PRICE
Medium (12") Hand Tossed Pizza **Whole:** Bacon, Chicken, Mushrooms Green Peppers	$16.95

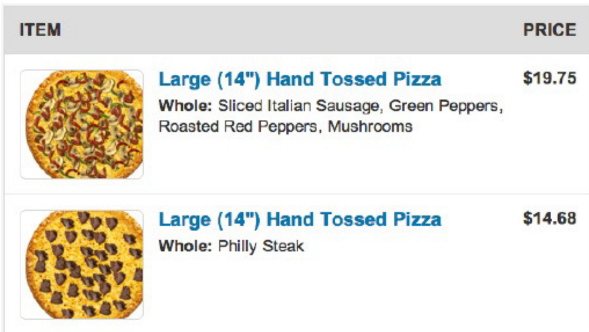
2. Write an equation you could use to determine the price of a pizza for a given number of toppings, then graph it. If you ordered your favourite medium pizza, how much would you expect to spend?

3. If you double the number of toppings that you order, do you pay twice as much for the pizza?

4. Write an equation for the price of a small pizza, and an equation for the price of a large pizza. Of the three sizes, which has the highest cost per topping, and why might this be? Of the three sizes, which has the lowest best prize?



ITEM	PRICE
Small (10") Hand Tossed Pizza **Whole:** Pepperoni	$9.99
Small (10") Hand Tossed Pizza **Whole:** Premium Chicken, Black Olives, Jalapeno Peppers	$11.99



ITEM	PRICE
Large (14") Hand Tossed Pizza **Whole:** Sliced Italian Sausage, Green Peppers, Roasted Red Peppers, Mushrooms	$19.75
Large (14") Hand Tossed Pizza **Whole:** Philly Steak	$14.68

5. Graph the equations for the small and large pizzas. If you had $20, what is the maximum number of toppings you could order in each size - small, medium, and large - and which ones would you choose?

Subject 36 **FUNCTIONS**

Please do not write on this exam paper and give it back at the end of the test

FALLING

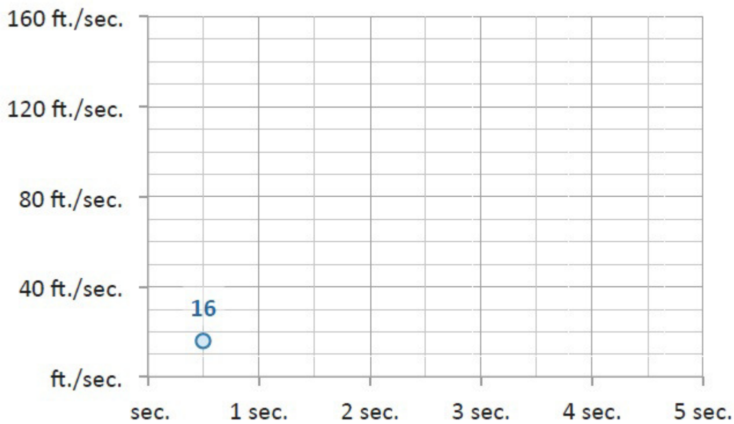
In this exercise, we'll use quadratic functions to explore the mathematics of how objects fall.

1. Ignoring air resistance, the function $d(t) = 16t^2$ gives the distance an object would travel in terms of its falling time in seconds . Using the annex, plot how far you expect an object would fall after 4 and 5 seconds, and graph the function d .

2. At the end of the popular musical *Les Misérables*, Inspector Javert falls from a bridge in the middle of Paris into the river below. As he falls, he sings...and sings...and sings...
...for 8 seconds! Using question 1, compute how high must the bridge have been, and does this seem reasonable?

3. Between 0 and 1 second, a falling object travels at an average speed of 16 feet per second (this means that at some point during the interval, it's travelling at *exactly* 16ft./sec. Let's assume this occurs at the halfway point).

Interval	Avg. Speed
0-1 seconds	16 feet/sec.
1-2 seconds	
2-3 seconds	
3-4 seconds	
4-5 seconds	



Plot the average speeds for the interval below, and write an equation to determine the speed of a falling object after t seconds. If Javert sang for 8 seconds while falling, how fast had he been travelling when he hit the water, and what do you think about your result?

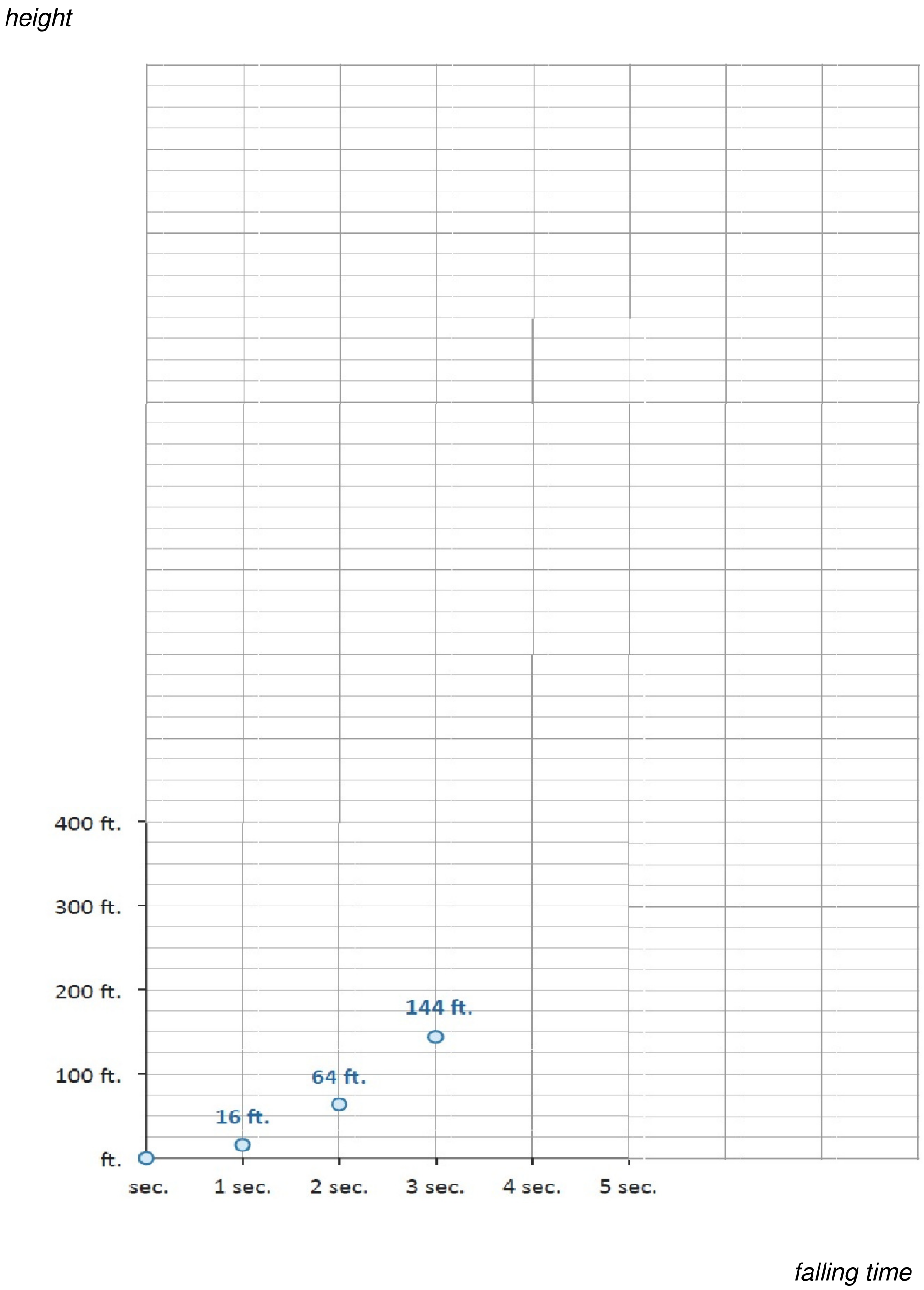
4. In 2015, Laso Schaller set a world record for the highest cliff jump: 58.8 meters, or around 193 feet. How fast Schaller had been travelling *in miles per hour* when he hit the water, and does this seem safe?

5. According to doctors, hitting the water at the speed of 25 mph can be fatal, even with special training. What is the maximum height you think the average person could safely jump from and why?

1 mile = 5280 feet
mph: miles per hour

Subject 36 **FUNCTIONS**

FALLING - ANNEX



Subject 37

FUNCTIONS

Please do not write on this exam paper and give it back at the end of the test

WAGES

1. In the United States, companies rely on hourly workers to prepare sandwiches, work as cashiers, etc... Imagine a business opens a new location and is hiring. The following table shows how many people would want to work at three different hourly wages (supply), and how many employees the company would want to hire (demand).

		\$3/hour	\$5/hour	\$12/hour
Supply:	# **individuals** who want job	72 people	120 people	288 people
Demand:	# employees **company** wants to hire	256 people	240 people	184 people

- Write an equation $s = a \times w$, where s is the supply and w the hourly wage: what is the value of a?
- Write an equation $d = a' \times w + b'$, where d is the demand and w the hourly wage: what are the values of a' and b'?

2. Governments often set a minimum wage; this is the least an employer can legally pay each hour. In the U.S., the federal minimum wage is \$7.25/hour. Graph the linear equations for the supply and demand you found in question 1. If the company only pays minimum wage, can it hire as many workers as it wants?

3. Find the lowest wage at which will enable the company to attract as many employees as it wants. Do you think this is the amount the company should pay its employees? Explain.

4. Millions of Americans work full-time at or near the minimum wage. Still, many of them struggle to afford rent, childcare, and other expenses, and rely on public assistance programs such as food stamps and Medicaid. To help these "working poor", some people think the government should raise the minimum wage from \$7.25/hour to \$15/hour. If this happened, how do you expect it would impact workers at the company?

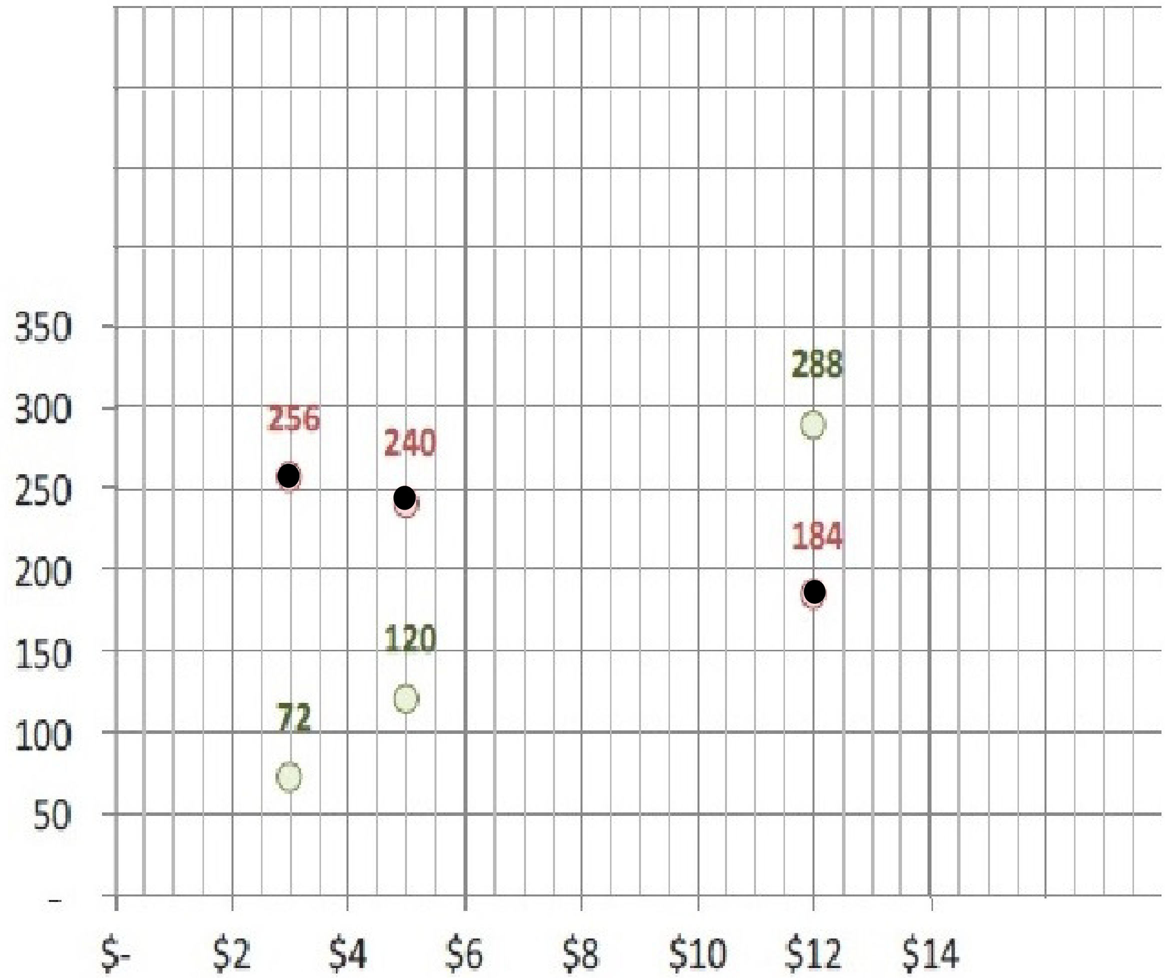
To hire: embaucher
To afford: réussir à payer
Wage: salaire

Subject 37 **FUNCTIONS**

Please do not write on this exam paper and give it back at the end of the test

WAGES - ANNEX

Number of workers



Wage

● *Demand*

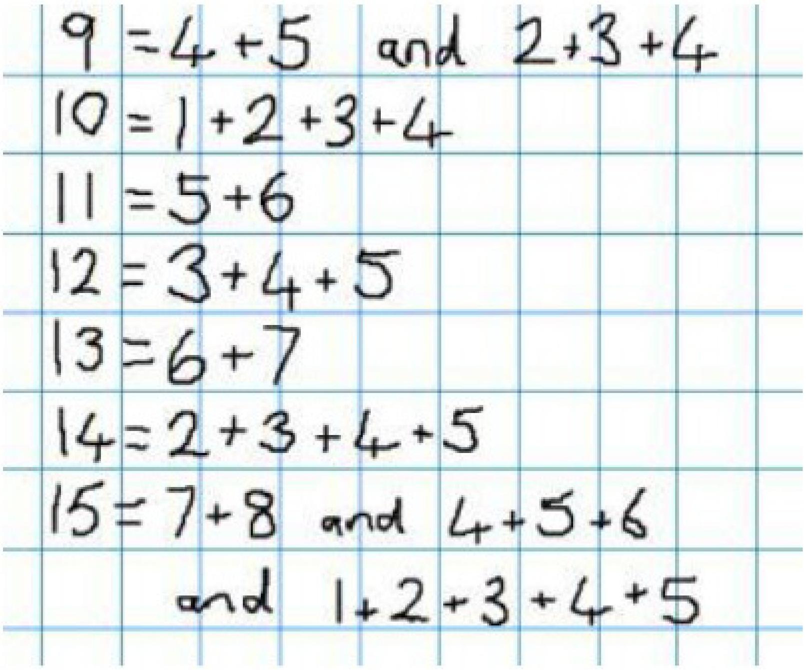
○ *Supply*

Subject 38 **SEQUENCES**

Please do not write on this exam paper and give it back at the end of the test

CONSECUTIVE NUMBERS

Charlie has been thinking about sums of consecutive numbers.
Here is part of his working out:



Alison looked over Charlie's shoulder:
"I wonder if we could write every number as the sum of consecutive numbers?"
"Some numbers can be written in more than one way! I wonder which ones?", Charlie says.
"9, 12 and 15 can all be written using three consecutive numbers.
I wonder if all multiples of 3 can be written this way?", Alison answers.
"Maybe you could write the multiples of 4 if you used four consecutive numbers...", Charlie suggests.

Try to answer all or some of the questions above. Explain your research, even if you didn't manage to find a solution.
Can you support your conclusions with convincing arguments or proofs?

NRich program, Cambridge university

Subject 39 **TRIGONOMETRY**

Please do not write on this exam paper and give it back at the end of the test

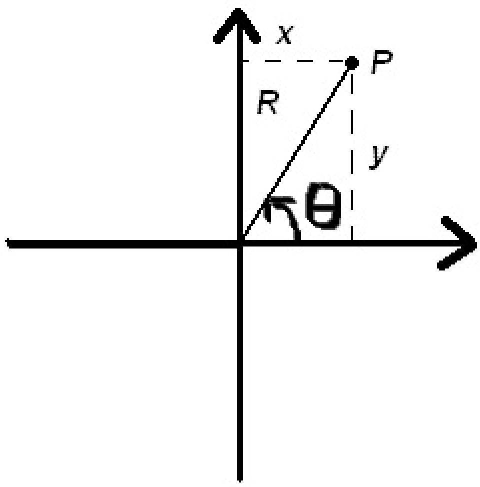
POLAR COORDINATES

The polar coordinates r (the radial coordinate) and θ (the angular coordinate, often called the polar angle) are defined in terms of Cartesian coordinates by

$$x = r\cos\theta \quad (1)$$
$$y = r\sin\theta, \quad (2)$$

where r is the radial distance from the origin, and θ is the counterclockwise angle from the x-axis. In terms of x and y,

$$r = \sqrt{x^2+y^2} \quad (3)$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right). \quad (4)$$



1: Why aren't polar coordinates unique?

2: Convert the polar coordinates (5 , 2.01) and (0.2 , 53 °) to rectangular coordinates to three decimal places.

3: Convert the rectangular coordinates (1 , 1) and (-2 ,-4) to polar coordinates to three decimal places. Express the polar angle t in degrees and radians.

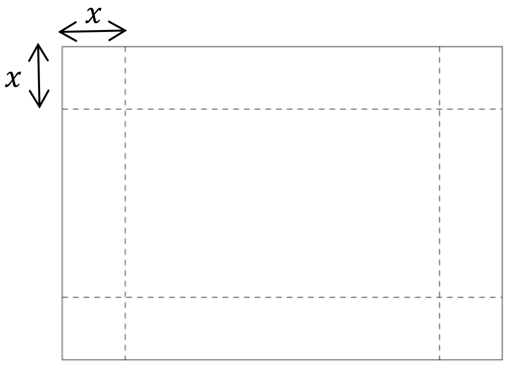
$\tan^{-1} = \arctan = \text{atan}$

Académie de Toulouse – sections européennes – session 2017
Sujet n°40

Please do not write on this document, and do not forget to hand it back at the end of the test.

Functions

Tom wants to build a box using a 14 inches by 10 inches sheet of metal.
To do so, he cuts out squares of side x out of each corner, then he folds the sheet to obtain an open box.



1. What are the values that x can take?
2. Show that the volume of the box can be expressed by:
$$V(x) = 4x^3 - 48x^2 + 140x$$
3. Find out the value of x that will maximize the volume of the box.
4. What is the maximum volume in cubic inches?
5. What is the maximum volume in cubic centimeters, knowing that 1 inch is 2.54 cm?

Académie de Toulouse – sections européennes – session 2017
Sujet n°41

Please do not write on this document, and do not forget to hand it back at the end of the test.

Functions

The following information can be found on the UK Government Website:

National speed limits

	Built-up areas mph (km/h)	Single carriageways mph (km/h)	Dual carriageways mph (km/h)	Motorways mph (km/h)
Cars, motorcycles, car-derived vans and dual-purpose vehicles	30 (48)	60 (96)	70 (112)	70 (112)

The objective of this exercise is to find out the braking* distance of a vehicle (also referred to as stopping distance), depending on the kind of road.

All along this exercise, we assume that:

- the car is travelling at the maximum authorized speed just before it brakes,
- $t = 0$ is the moment when the driver brakes,
- the car acceleration when the driver brakes is $-5.5\ m.s^{-2}$.

1) Braking in built-in areas:

a) Knowing that $a(t) = v'(t)$, which means that the car acceleration, $a(t)$, is the derivative of the car velocity $v(t)$:
Work out the expression of $v(t)$ after the car brakes.

b) Knowing that $v(t) = s'(t)$, which means the car velocity, $v(t)$, is the derivative of the distance $s(t)$ it has travelled:
Work out the expression of $\mathrm{s(t)}$ after the car brakes.

c) How long does the car need to stop?

d) Work out the braking distance.

2) **Braking on motorways:**
You'll answer the same questions as above for a car travelling on a motorway.

3) **Speed and braking distance:**
According to your previous results, is braking distance proportional to velocity?

NB: to brake = freiner

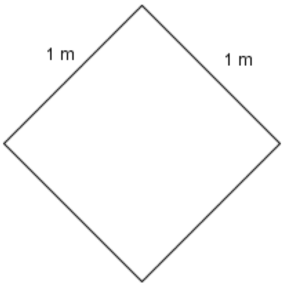
Académie de Toulouse – sections européennes – session 2017
Sujet n°42

Please do not write on this document, and do not forget to hand it back at the end of the test.

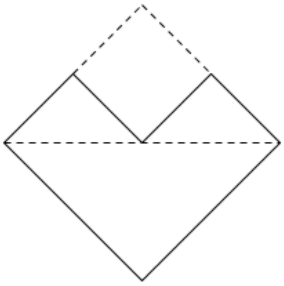
Sequences

In Northern Canada, a woodcutter has just broken his saw while cutting an enormous tree. Luckily, he had brought in his car truck a sheet of metal, which is a square whose sides are one meter long.
He will have to manage and use it to build a new saw.

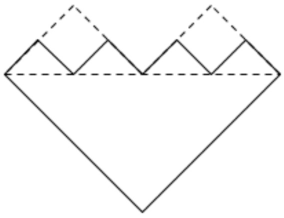
After the first cut, he gets two identical sawteeth.
Then, at each of the following cutting, he cuts every sawtooth into two sawteeth.



Initial sheet



1st cut



2nd cut

The woodcutter makes 6 cuttings before he decides his new tool is finished.
Before the saw is operational, he has to sharpen every sawtooth.

1. How many teeth does the new saw have?
2. How long is the side of each tooth? You'll round to the millimeter.
3. What is the total length of metal the woodcutter will have to sharpen?
4. What would be the total length of metal to sharpen for 10 teeth? 15 teeth?

Académie de Toulouse – sections européennes – session 2017
Sujet n°43

Please do not write on this document, and do not forget to hand it back at the end of the test.

Sequences

Reverend Thomas Robert Malthus (13 February 1766 – 29 December 1834) was an English cleric and scholar, influential in the fields of political economy and demography.
Malthus observed that an increase in a nation's food production improved the well-being of the population, but the improvement was temporary because it led to population growth.

In 1800 in England, there were 8.3 million inhabitants, who were mainly poor people, but English agriculture produced exactly the right quantity of food so that everyone got enough to eat.
Thomas Malthus assumed that:
- English agricultural production enabled to feed 400,000 additional inhabitants every year,
- English population was growing by 2% every year.

1. In 1801, did all English people have enough to eat, according to these predictions?

2. For any whole number n, let u_n be the number of English inhabitants, in millions, that could be fed by English agriculture in year 1800+n.
   a) Compute u_0, u_1 and u_2.
   b) Justify that (u_n) follows an arithmetic progression, whose first term and common difference you will specify.
   c) For any whole number n, express u_n in terms of n.

3. For any whole number n, let v_n be the number of inhabitants in England in year 2010+n, in millions.
   a) Compute v_0, v_1 and v_2 (rounded to the nearest thousand).
   b) What type of progression is (v_n) sequence?
   c) For any whole number n, express v_n in terms of n.

4. Find out the first year when there wasn't enough food for all English people, according to Malthus' predictions.

Académie de Toulouse – sections européennes – session 2017
Sujet n°44

Please do not write on this document, and do not forget to hand it back at the end of the test.

Statistics

Here is an extract from “Erasmus + Programme – Annual Report 2015” released by the European Commission.

In 2015, a total of 159 higher education projects were funded under a strategic partnerships action. Compared to 2014, the numbers of both applications and selected projects increased slightly.

HIGHER EDUCATION	CALL 2014	CALL 2015
Projects received	917	988
Projects granted	154	-
Success Rate	-	16%
Grant (in million EUR)	42.02	43.43

1. Fill in the two cells that have been deleted.
2. What is the percentage of increase of the number of projects that were granted between 2014 and 2015?
3. Work out the average grant per granted project in 2014 and in 2015 (rounded to the nearest thousand euros).
4. By what percentage did the total grant increase / decrease between these 2 years?
5. Assuming this annual rate will be constant in the coming years, what will be the grant for higher education projects in 2020 (rounded to the nearest ten thousand euros)?

Académie de Toulouse - sections européennes - session 2017

Subject n°18

PROBABILITIES

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

The digits of a biased six faces die are labelled 1, 2, 3, 4, 5 and 6. Now Barbara is playing dice.

The probabilities of several dice rolls are given in the table below.

Number	1	2	3	4	5	6
Probability		0.14	0.34	0.12		

The probability to get a 1 or a 5 is the same. The probability to get a 6 is twice as much to get a 5.

1. Calculate the missing probabilities.
2. What's the probability that Barbara gets an even number?
3. Knowing the result is an odd number, what's the probability she obtained a 3?
4. Barbara throws the die 200 times. How many times should she expect to get a 4?
5. If she throws the die a very large number of times, and if she calculates the average of the results, what should she expect?

<u>Vocabulary:</u> a biased die: un dé pipé.

Subject n°19

PROBABILITIES

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

The table below shows the probabilities of selecting raffle tickets from a box. The tickets are coloured red, white or blue and numbered 1, 2, 3 or 4.
A raffle ticket is taken at random from the box.

Coloured tickets		**Number**			
		1	**2**	**3**	**4**
Colour	**Red**	0.08	0.04	0.06	0.10
	White	0.18	0.14	0.12	0.02
	Blue	0.06	0.16		0.00

Calculate the probability that:

1. It is blue and numbered 3.
2. It is numbered 1.
3. It is red.
4. It is either white or numbered 2.
5. It is neither blue, neither numbered 4.
6. It is white knowing it is numbered 2.
7. There are 500 tickets in the box. How many both blue and numbered 4 tickets do you expect to find?

<u>Vocabulary:</u> raffle: tombola.

Subject n°20

SEQUENCES

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

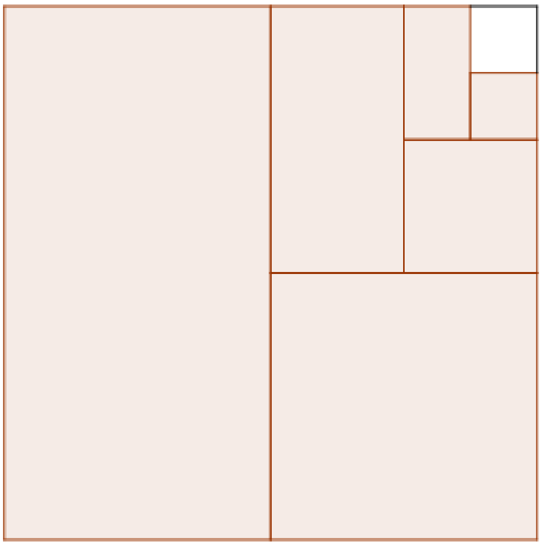
1. Describe the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

2. A new sequence S(n) is created by adding together the first n terms of the previous sequence.
   $S(1)=\frac{1}{2}$, $S(2)=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ etc…
   Write down the first five terms of the sequence S(n).

3. Give the expression of S(n) with respect to n.

4. a. What's the link between the diagram below and the sequence S(n) ?

   b. Hence, find out the limit of the sequence S(n).



   You can also do it using another method, which one?

5. Calculate the first value of n such as S(n) > 0.999.

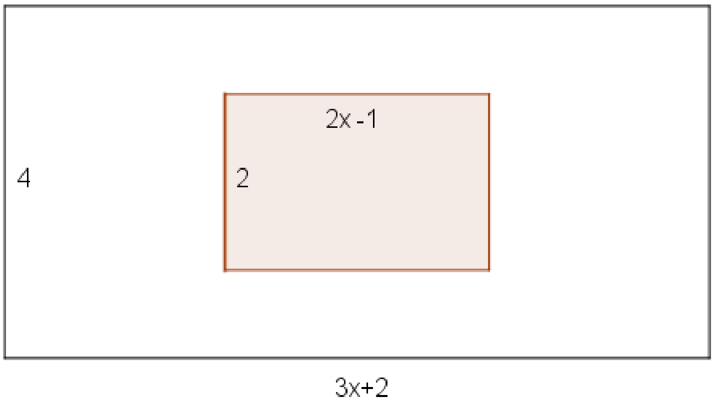
Académie de Toulouse - sections européennes - session 2017

Subject n°21

FUNCTIONS

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

1. Show that the **not shaded area** of this rectangle is $8x + 10$



2. As $2x - 1$ is the length of the inside rectangle, what should the minimum value for x be?

3. Write, with respect to x, the expression of f(x), where f(x) is the ratio of the **not shaded area** by the whole area of the figure.

4. Explain what f(x) tends to when x gets very large.

5. Determine f'(x) the derivative of f and deduce the variations of the function f.

6. Find the value of x such as f(x) = 0.9.

Académie de Toulouse - sections européennes - session 2017

Subject n°22

FUNCTIONS

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

The formula for the stopping distance, S, in meters, for a vehicle travelling at v km/h is:

$$S = \frac{v^2}{180} + \frac{v}{1.8}$$

1. A car is travelling at 100 km/h. How long is the stopping distance?

2. The speed of a school bus is 25 m/s. Calculate the stopping distance.

3. A car and a truck are travelling head-on towards each other on a narrow road. They see the danger coming, and start to brake at the same instant.
The car is travelling at 70 km/h and the truck at 52 km/h.
Calculate the minimum distance between them if they just stop in time before the accident.

4. Find the speed of a vehicle which has braked for 60 m before stopping.

5. In fact, the driver's stopping distance is the sum between the distance of reaction and the braking distance. In the formula given above, $\frac{v^2}{180}$ is the braking distance, and $\frac{v}{1.8}$ is the distance of reaction. We usually assume that the time of reaction is about 2s. Therefore, can you explain the formula $\frac{v}{1.8}$ for the distance of reaction?