Académie de Toulouse – sections européennes – session 2020

Subject n°1

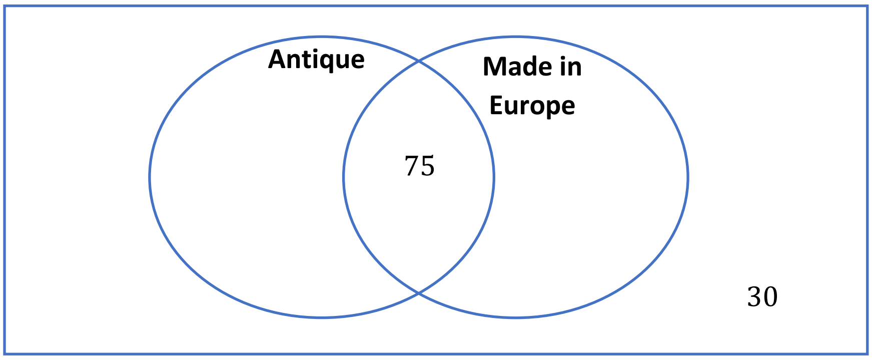
Probability

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

A museum has a collection of 200 crockeries[1].

The two-way table and the Venn diagram show some information about the crockeries.

	Made in Europe	Made outside Europe	Total
Antique			
Not antique			
Total		80	200



1. Complete the table and the Venn diagram. Justify your results
2.
   a. Find the probability that a randomly chosen piece of crockeries is made in Europe.
   b. Find the probability that a randomly chosen piece of crockeries is antique and made in Europe.
   c. Find the probability that a randomly chosen piece of crockeries is antique given that it is made in Europe.

[1] Vaisselle

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Subject n°2

Reasoning and arithmetic

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

1. **Proof**
   a. Prove that the difference between the squares of two consecutive even numbers is always a multiple of 4.

   b. Prove that the product of an odd number and an even number is even.

   c. Prove that the sum of any three odd numbers is odd.

2. **True or false?**

   "The difference between any two consecutive square numbers is always a prime number."

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Subject n°3

Sequences

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

In order to employ someone, a company offers two different pays.

→ **Pay 1:** an initial monthly pay of £1,100 and a monthly increase of £150 per year.

→ **Pay 2:** an initial monthly pay of £1,100 and a monthly increase of 10% per year.

Question: Which pay is the best if the employee stays 10 years in the company?

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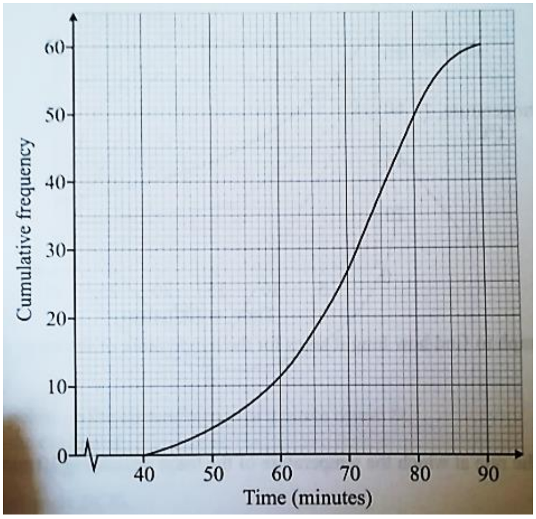
Subject n°4

Statistics

Please, do not write on the exam paper and do not forget to give it back at the end of the test.

Some teams took part in a charity race in 2020.

The cumulative frequency graph shows the time that the teams took to complete the course.



1. How many teams took part to the race?
2. How long does it take to complete the race:
   a. For the best team?
   b. For the worst team?

The table below summarises the time that teams took to complete the race in 2019.

2019 Race times	
Median	76 minutes
Interquartile range	18 minutes
Winning time	37 minutes

3. Compute the difference between the best time in 2019 and 2020?
4. On average were the teams faster in 2019 or 2020? Explain your answer.
5. Was the race duration more consistent in 2019 or 2020? Explain your answer.

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<u>Subject n°5</u>

MCQ

Please do not write on the exam paper, and do not forget to give back the examination paper at the end of the test.

This exercise is a Multiple Choice Questionnaire.

For each question, find out the **only one** possible answer and justify it :

1. How many minutes would it take a car to travel 42 miles at a constant speed of 56 miles per hour ?

a) 90 **b)** 80 **c)** 45 **d)** 40 **e)** 30

2. A rectangle has a perimeter of 16 meters and an area of 15 square meters. What is the longest of the side lengths, in meters, of the rectangle ?

a) 3 **b)** 5 **c)** 10 **d)** 15 **e)** 16

3. In the standard (*x* ; *y*) coordinate system, what is the slope of the line segment joining the points (3 ; – 5) and (7 ; 2) ?

a) $\frac{-7}{4}$ **c)** $\frac{-3}{4}$ **d)** $\frac{4}{7}$ **e)** $\frac{3}{4}$ **f)** $\frac{7}{4}$

4. Among the marbles in a jar, 14 are orange. Sarah randomly takes one marble out of the jar. If the probability is $\frac{2}{7}$ that the marble she chooses is orange, how many marbles are there in the jar ?

a) 4 **b)** 14 **c)** 28 **d)** 49 **e)** 98

5. If *b* is a positive integer that divides both 98 and 140, but divides neither 20 nor 35, what should you get when you add the digits in *b* ?

a) 1 **b)** 3 **c)** 4 **d)** 5 **e)** 6

6. A right circular cylinder has a base diameter and height of 10 inches each. What is the total surface area of this cylinder, in square inches ?

a) 25π **b)** 50 π **c)** 75 π **d)** 150 π **e)** 200 π

<u>Vocabulary</u> : slope : *coeffiecient directeur* ; marble : *boule*

Subject n°6

Probabilities

Please do not write on the exam paper, and do not forget to give back the examination paper at the end of the test.

500 new vehicles have been sold by a garage sale last year.

85 are ethanol vehicles.

13 % are electric vehicles.

7.8% are hybrid vehicles (both electric and ethanol).

1. Complete the following table :

	Ethanol	**Not ethanol**	**Total**
Electric			
Not electric			
Total			500

2. The type of a vehicle is chosen at random.

a) What is the probability that this vehicle doesn't run on ethanol ?

b) What is the probability that this vehicle runs on ethanol but not on electricity ?

c) Knowing that the chosen vehicle runs on electricity, what is the probability that this vehicle also runs with ethanol ?

3. Paul buys a hybrid car in this garage.

He chooses a model costing £27,000. He negotiates a discount of 3.5 % on the price of his new car and sells back his old car for £1,250.

How much will Paul pay for his new car ?

Vocabulary : form = fiche

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Subject n°7

Sequences

Please do not write on the exam paper, and do not forget to give back the examination paper at the end of the test.

An important industrial company discharges 50,000 tonnes of waste in 2019.

To respect new antipollution norms, it must reduce this quantity to less than 30,000 tonnes in 10 years.

The company undertakes to reduce the quantity of waste of 4 % per year.

1. If the company produces 48,000 tonnes of waste in 2020, is the undertaking respected ?

2. Let r_n be the number of tonnes discharged in year 2019 + n.
a) Show that (r_n) is a geometric sequence. What is its common ratio ?
b) Express r_n in terms of n.

3. Compute the quantity of waste expected in year 2029. Round your answer to the nearest tonne. Will the undertaking be respected in 2029 ?

4. Would an annual rate of decrease of 5 % allow to respect the undertaking ?

Vocabulary : undertake = prendre l'engagement discharge = rejeter

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<u>Subject n°8</u>

Statistics

Please do not write on the exam paper, and do not forget to give back the examination paper at the end of the test.

Part I :

In a sale, the marked prices are reduced by 30%.

a. Calculate the sale price of a jacket if the marked price is £350.
b. Find the marked price of a dress if the sale price is £168.

Part II

The fraud department studied a sample made of 200 boxes supposed to contain 170 g of cheese. The table below gives the results :

Weight (g)	166.5	168	168.5	169	169.5	170	170.5	171	171.5	172
Frequency	1	6	12	21	36	48	34	18	14	10

a. Compute the mean (average) and the median of this data set.

b. Calculate the range and the interquartile range.

c. The regulations require a median of 170 g and an interquartile range of 1.35 g. Does this sample meet the requirements ?

d. The company changes the setting of weighing machines when more than 20 % of the boxes weigh 171 g or more. Should it do so ?

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Subject n°9

FUNCTIONS

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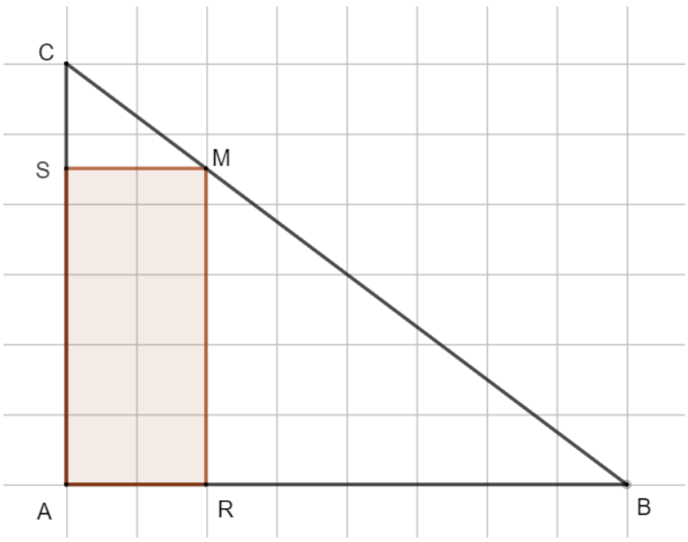
Optimization

We consider a right angle ABC triangle, such as AB = 8 cm and AC = 6 cm.



For every moving point on the [BC] line segment, we build the ARMS rectangle as shown on the geometric shape above.

Let x be equal to the length AR. Let *x* be the length of line segment AR in cm, and *f*(*x*) the area of ARMS rectangle in cm².

1. Which interval does x belong to?
2. Make a conjecture about the variations of the f function.
3. Using Thales Theorem, prove that $f(x) = -\frac{3}{4}x^2 + 6x$.
4. Using your own calculator, try to estimate the maximal area of the ARMS rectangle.
5. Now, using your own math skills, prove your previous result.
6. For which values of x is the rectangle area greater or equal to 9?

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Subject n°10

PERCENTAGES

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Different rates of evolution

The quotation of the famous Ladybug car (now collector, but common in the sixties), increases by 8 % every year.

1. What's the percentage of increase after two years?
2. Imagine that once happen a fall in the quotation. What should be the percentage of decrease in order to recover the former price. In other words, calculate the reciprocal evolution of an increase by 8%.
3. Assuming that the rate of evolution will always be the same, and equal to 8%, calculate, using your own calculator, the number n of years necessary to multiply the initial price by more than 10.
4. Justify that it's possible to answer the previous question by solving the inequation $1.08^n \geq 10$.
5. What should be the rate of evolution if we'd like to see the price multiplied by 10 in 20 years?

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Subject n°11

SEQUENCES

Please, do not write on the exam paper
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The study of an evolution

As he is 1.75 m tall and weighs 120 kg, Brian is too fat. So, on the advice of his doctor, he's been beginning a diet in order to slim, on January 1st, 2020. Every month he loses 5% of the weight he had the first of the month, but, on the last day of the month, he breaks down, eats too much, and he takes back two kg.
We denote p_n Brian's weight after n months of such a diet. So p_0 = 120 kg.

1. Calculate p_1 and p_2.
2. Justify that for any $n : p_{n+1} = 0.95 \times p_n + 2$.
3. Explain why (p_n) is neither a geometric sequence, neither an arithmetic sequence.
4. The Body Mass Index (BMI) is the ratio of the weight (in kg) by the square of the size (in meters). What is Brian's BMI when he begins his diets?
5. You're considered overweight when your BMI is more than 25. What should be the maximal weight for Brian in order not to be overweight anymore?
6. If he goes on with his diet, calculate when he may reach his goal.
7. Now we assume that for any n: $p_n = 80 \times 0.95^n + 40$.
   Using this formula, try to find again the result of question 5.
8. What do you think of the Brian's weight tendency on a very long time?

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Subject n°12

STATISTICS

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Marks in a class

The scores achieved by 16 students on a math test are as follows:
2; 6; 7; 8; 9; 9; 10; 10; 11; 11; 13; 14; 14; 14; 16; 19

1. Find the mean, the mode, the median and the quartiles of this set of data.
2. Which proportion of the students represents the ones with a mark equal or greater than 13? On a pie chart, what should be the corresponding angle, in degrees?
3. Four students did not turn up for the test and were awarded a mark of zero.
   Find the new mean, mode, median, and quartiles of this new set of data.
4. Draw the two box plots associated with these two sets of data.
5. Which of the mean or the median is the least appropriate measure of location for the test score of the whole class of 20 pupils?
6. This test had a weighting coefficient of **two**. The teacher wants to give a chance to the students to have a better average, with a new re-take test, but with a weighting coefficient of only **one**. What should be the new mean (average), at the re-take exam, in order to get a mean of 11 for the whole class, on all of the two assessments?

Subject n° 13 FUNCTIONS

Please do not write on this document, and hand it back to the jury at the end.

Ex 1 :

The motion of an object can be simply described by a graph of the distance travelled against the time taken.

Match each motion to the correct graph.

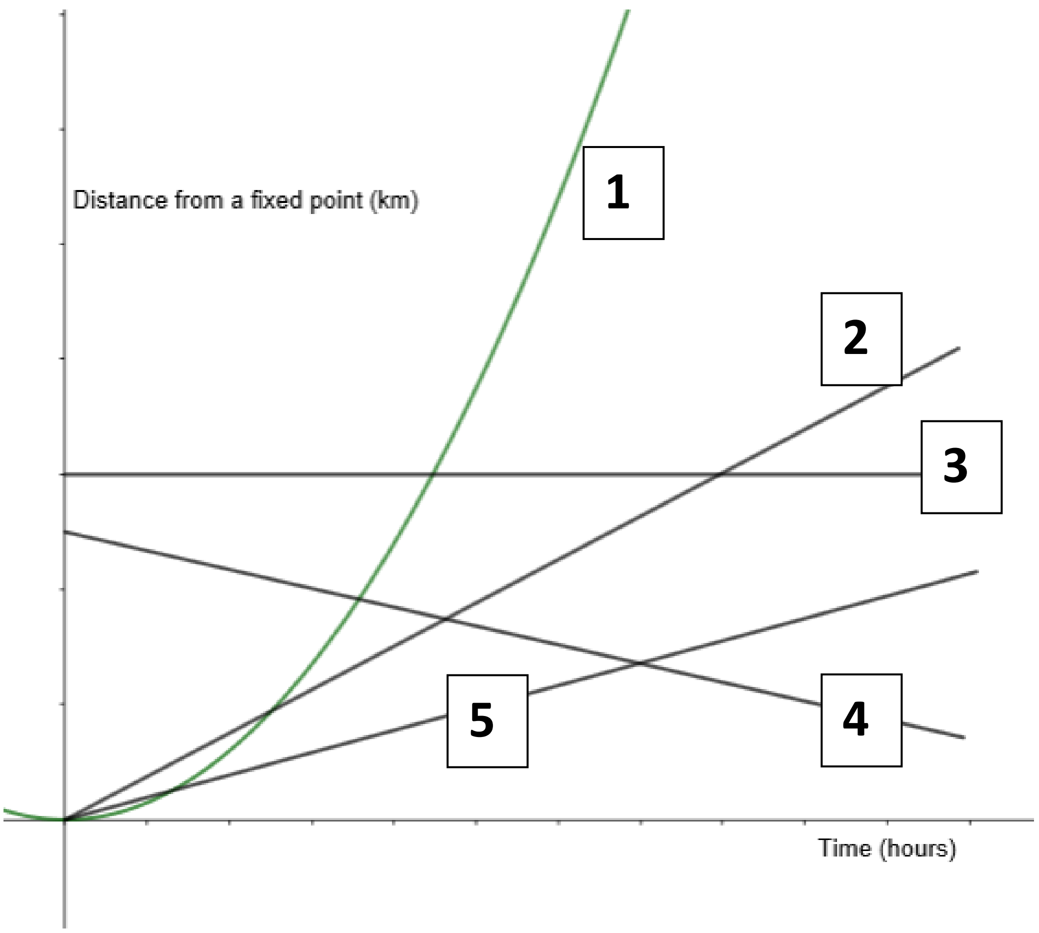
A : Travelling away slowly at constant speed

B : Travelling away fast at constant speed

C : Travelling away with increasing speed

D : Travelling back at constant speed

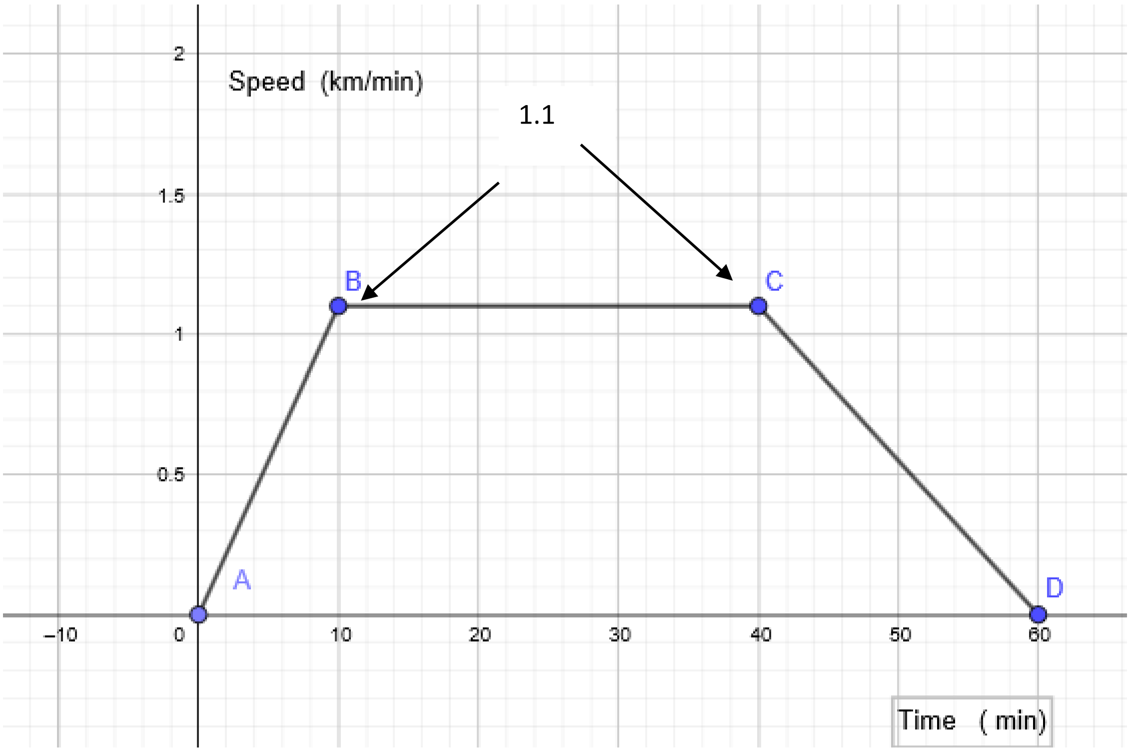
E : Not moving



How can you interpret the intersection between two curves, in this graph ?

Ex 2:

A car changes speed as shown in this speed–time graph.



1. For each part of the graph (from A to B, from B to C, form C to D), describe the motion of the car .

2. How can you interpret the area under the graph between B and C, in that context ?

3.The car breaks down at OXFORD. Considering this graph : which letter(s) could be associated with Oxford ?

4. Over the whole journey (excluding stops), what was the average speed (km /min) of the car?

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Subject n° 14   STATISTICS

Please do not write on this document, and hand it back to the jury at the end.

Sets can be shown in a diagram called a Venn diagram after the English mathematician John Venn (1834–1923). The members of the set are shown within a closed curve.

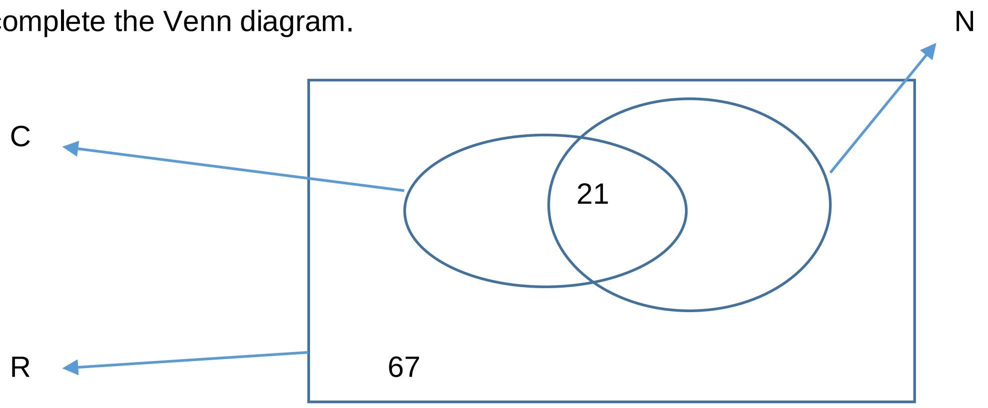
On the Venn diagram below,

R is the set of the ice creams I serve in my restaurant

C , the set of the ice creams containing chocolate : we have 53 such ice creams

N , the set of the ice creams containing nuts. : we have 36 of them.

a. Copy and complete the Venn diagram.



b. How many ice creams are there in the restaurant?

c. How many ice creams contain chocolate but no nuts?

d. How many ice creams contain chocolate or nuts, but not both?

e. Describe the set N ∩ C in words.

f. Describe the set N ∪ C in words. Is the equality N ∪ C = C ∪ N true?

g. Consider the set B : the set of the ice creams containing bananas

If (B ∩ C) = ∅, what can you say ?

Subject n°15

PROBABILITY

Please do not write on this document and do not forget to hand it back to the jury at the end of the test.

The following system of categorizing social class is widely used in Great Britain:

- Class A: Higher managerial, administrative and professional
- Class B: Intermediate managerial, administrative and professional
- Class C1: Supervisory, clerical and junior managerial, administrative and professional
- Class C2: Skilled manual workers
- Class D: Semi-skilled and unskilled manual workers
- Class E: State pensioners, casual and lowest grade workers, unemployed with state benefits only.

	Class A	Class B	Class C1	Class C2	Class D	Class E
Share in population	4%	23%	28%	20%	15%	10%

Source : https://en.wikipedia.org/wiki/NRS_social_grade

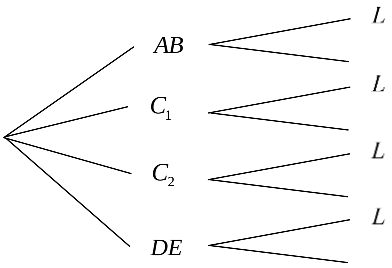
An survey institute conducted a survey to determine the influence of social class on the vote for Brexit, where voters only had to choose between Remain and Leave (the European Union).

	Classes A and B	Class C1	Class C2	Classes D and E
Vote for *Leave*	41%	48%	62%	64%

Source : https://www.ipsos.com/ipsos-mori/en-uk/how-britain-voted-2016-eu-referendum

We randomly choose a voter among all respondents in the survey. Let us consider the following events:

- L : “the respondent votes for Leave”;
- AB : “the respondent belongs to class A or B”;
- C_1 : “the respondent belongs to class C1”;
- C_2 : “the respondent belongs to class C2”;
- DE : “the respondent belongs to class D or E”.



1) Reproduce and complete the probability tree using all the above data.

2) Give the probability that the respondent is a skilled manual worker voting for Remain.

3) Give the probability that the respondent is a leave voter.

4) If the respondent decides to vote for Leave, what is the probability that he belongs to class A or B ?

5) *Subsidiary question :* The union of classes A, B and C1, called ABC1, is often (but wrongly) identified as the middle class and the class C2DE as the working class. If the respondent decides to vote for Leave, what is the probability that he belongs to the middle class?
You may use the fact that: $(AB \cup C_1) \cap L = (AB \cap L) \cup (C_1 \cap L)$.

Vocabulary: Respondent: enquêté(e)

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Subject n°16

SEQUENCES

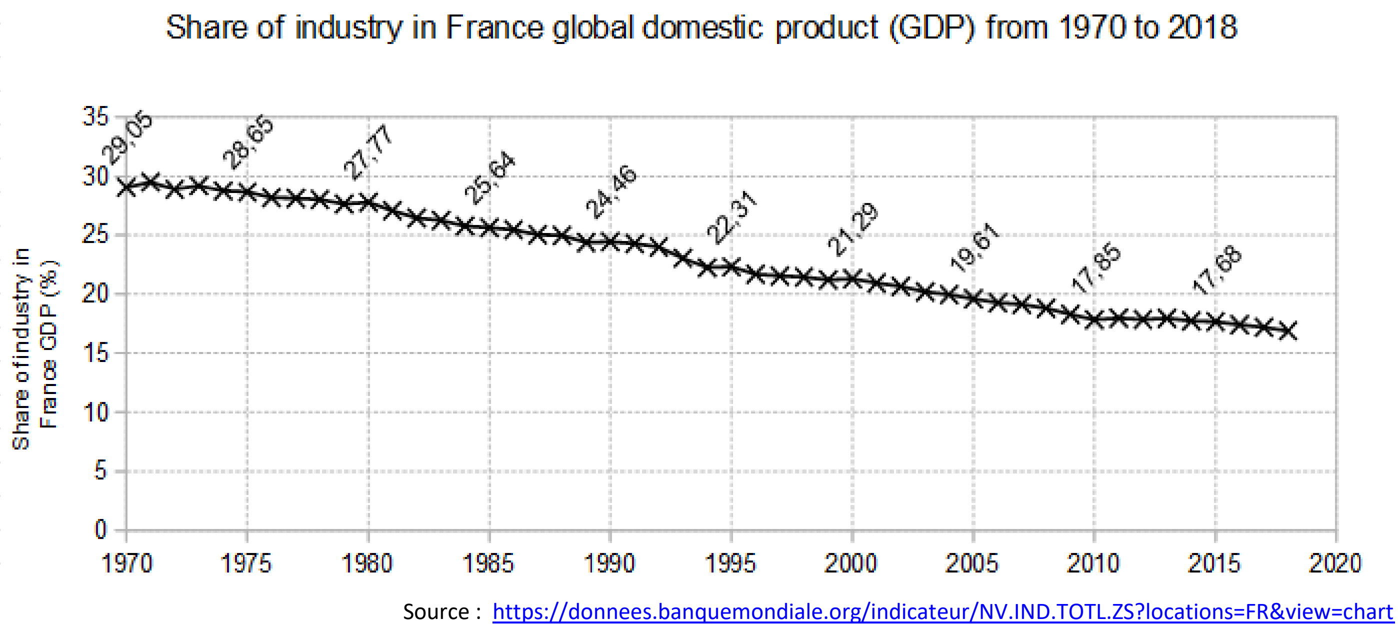
Please do not write on this document and do not forget to hand it back to the jury at the end of the test.

The Gross Domestic Product (GDP) is the total value of goods produced and services provided in a country during one year. It is divided between Industry, Fabrication, Agriculture and Services. Many European countries experience a deindustrialization: a reduction in the number of jobs in the industrial sector and a reduction of the share of the industrial sector compared to other sectors of activity.

> *The spectacular rise in power of the emerging countries of East Asia, Latin America or Eastern Europe during the years 1990-2000 further aggravated the deindustrialization, as it pushed French industrial companies to reorganize their production on a global basis (for instance, the European Union), to face this unforeseen competition by its scale and its speed. The acceleration of deindustrialization thus appeared as a consequence of the opening of industrialized countries to the World, the corollary of which was the global and unprecedented competition of employees with each other. The number of employees rose from 1.5 billion worldwide in the early 1980s to more than 3 billion in 2010.*

Source (en français) : https://journals.openedition.org/rge/6333#tocto2n2

The following plot shows the evolution of the share of industry in France gross domestic product between 1970 and 2018.



Source : https://donnees.banquemondiale.org/indicateur/NV.IND.TOTL.ZS?locations=FR&view=chart

An economist models the GDP of the year $1970+n$ by $u_n = 29{,}817 - 0{,}283n$ for every $n \geq 0$.

1) Compute the terms of the sequence corresponding to 2005, 2010 and 2015 and compare with the real values of the GDP. Do you think the model accurately reflects reality?
2) Prove that the sequence (u_n) is arithmetic. What are its characteristics?
3) Using that model, what dividends could the investor expect for the year 2025?
4) What is the limit of the sequence (u_n)? Comment on the model.

Vocabulary: Gross Domestic Product : Produit Intérieur Brut

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Subject n°17

Probabilities

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In York, it's rainy one third of the days.
Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$.
If it's rainy and there is heavy traffic, Jenny arrives late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it is not rainy and there is no heavy traffic.
In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25.
You pick a random day.

1) Model this situation with a probability tree.

2) What is the probability that it's not raining and there is heavy traffic and Jenny is not late ?

3) Show that the probability that Jenny is late is $\frac{11}{48}$.

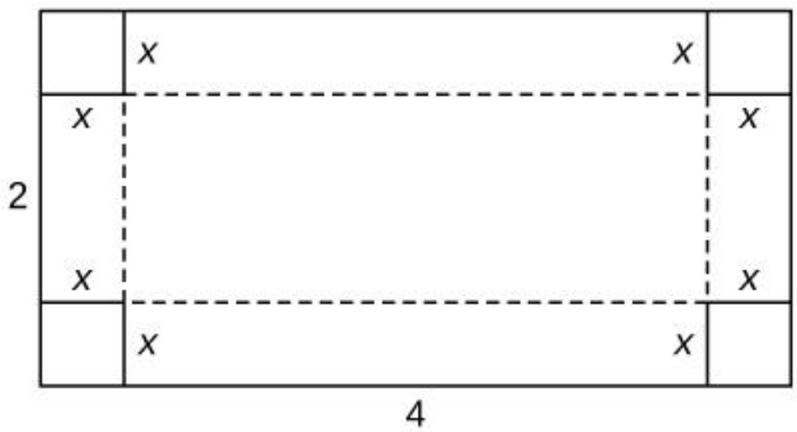
4) Given that Jenny arrived late at work, what is the probability that it rained that day ?

Subject n°18

Functions

Please do not write on this document, and do not forget to hand it back to the jury at the end.

You are constructing a cardboard box with the dimensions 2 m by 4 m.
You then cut equal-size squares from each corner so you may fold the edges



1) Justify that the volume of this box in terms of x is V(x) = $4x^3 - 12x^2 + 8x$.

2) Determine the derivative function V' of the function and its sign. Deduce the variation table of V(x).

3) What are the dimensions of the box with the largest volume?

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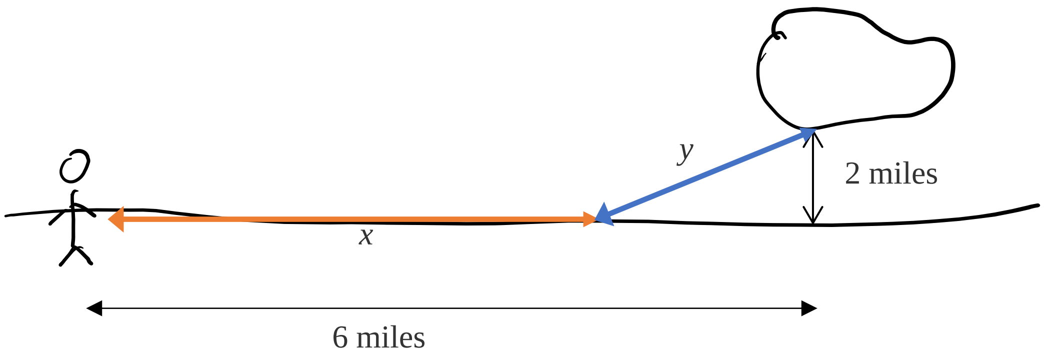
Subject n°19

Functions

Please do not write on this document, and do not forget to hand it back to the jury at the end.

An island is 2 miles away perpendicular from the shoreline
Tom is staying 6 miles away along the shoreline. He is planning to go to the island by running and then swimming.
Let's assume that Tom runs at a rate of 8mph and swims at a rate of 3mph.



You now have to determine how long should Tom run before swimming to minimize the time it takes to reach the island ?

Let's assume Tom runs *x* miles along the shoreline and then swims *y* miles to reach the island.

1) Show that the total time to reach the island depending on *x* is $T(x) = \frac{x}{8} + \frac{\sqrt{x^2-12x+40}}{3}$.

2) Determine the derivative function T' of the function and its sign.

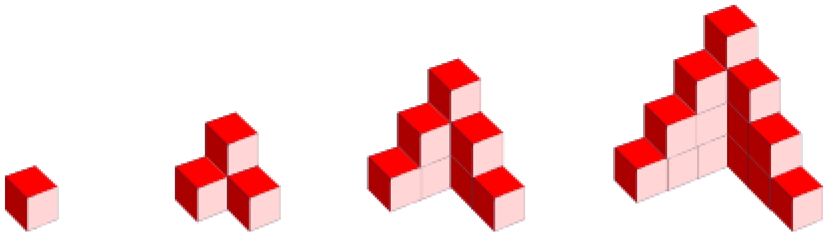
3) Prove that Tom needs to run approximately 5.2 miles before swimming to minimize the time to reach the island.

Subject n°20

Sequences

Please do not write on this document, and do not forget to hand it back to the jury at the end.

Here is a picture of four models numbered (1), (2), (3) and (4). Some of the cubes are hidden behind other cubes.



(1) (2) (3) (4)

Model (1) consists of one cube.
Model (2) consists of four cubes and so on.
We label u_n the needed number of cubes to build the model (n). So $u_1 = 1$ and $u_2 = 4$.

(1) How many cubes are in the model (3) or what is u_3 ?

(2) How many cubes are in the model (4) or what is u_4 ?

(3) If a model (5) was built, how many cubes would it take or what is u_5 ?

(4) Compute the difference between two consecutive terms : $u_2 - u_1$; $u_3 - u_2$ and $u_4 - u_3$
Deduce an expression of $u_{n+1} - u_n$ in terms of n.

(5) Find an expression for the number of cubes used in the n^{th} model.

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Subject n°21

Algebra and functions

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Adolf Eugen Fick (1829 – 1901) was a German-born physician and physiologist. In 1855, he introduced Fick's laws of diffusion, which describe the diffusion of a gas across a fluid membrane. Assuming that they can also describe the diffusion of heat through meat, these laws can be applied to cooking.

Let's suppose a turkey is the same as a sphere with weight M.
The following equation, falling out from Fick's laws will give the cooking time for the Christmas turkey :

t : time in min

M : weight in kg

$$t = \left(\frac{M}{k_0}\right)^{\frac{2}{3}} T_0$$

k_0 : coefficient

T_0 : temperature of the oven in °C

1) Given that the previous relation is equivalent to $\left(\frac{t}{T_0}\right)^3 = \left(\frac{M}{k_0}\right)^2$, write k_0 in terms of t, M and T_0 .

2) Peter Barham, a physicist and gourmet from Bristol found the following results :
At 180°C, a 5kg turkey needs two hours and twenty five minutes for cooking and a 7kg turkey needs three hours .
Find the approximate value of k_0 rounded to 1 d.p. , then deduce the cooking time for a 6kg turkey.

3) Additional data :
At 160°C, a 5kg turkey needs three hours and thirty five minutes for cooking and a 7kg turkey needs four hours and thirty minutes.
What can you deduce ?

4) Sketch a graph to model time as function of M, at 160°C and 180°C, then comment on it.

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Subject n°22

Miscellaneous exercises

Please, do not write on this document and do not forget to hand it back to the jury at the end of the test.

<u>Exercice 1 :</u>

Every morning, Sherlock leaves his house. Sometimes he takes his magnifying glass. If an investigation is underway, he always takes his magnifying glass, else he takes it only twice out of five. On average, he is concerned with an investigation one morning out of three.

Today, Doctor Watson just joined him in Piccadilly with a crypted message found under his door ; what is the probability he could use his magnifying glass for decrypting?

<u>Exercise 2 :</u>

In Marseille hospital, a patient is waiting for a heart transplantation.
The helicopter, carrying the new heart and the medical team, took off from Clermont-Ferrand at 9.30 am. For the surgery to take place under proprer conditions, the helicopter must land in Marseille at 12.00 at the latest. Marseille is about 330 km from Clermont-Ferrand, as the crow flies[1].

1) What should be the minimal speed of the helicopter ?

2) Weather conditions are really bad, the pilot is worried.
Take the speed found in question 1) as an average speed for the trip. Give the expression of the distance travelled from Clermont-Ferrand, D in km, with respect to the duration of the flight, t in hours.

3) The helicopter passes over Avignon at 11:05 a.m., it is 214 km from the start.
Will it be on time in Marseille ?

Information *:* Between the time a heart is removed and the time it is transplanted, it should not exceed 3 or 4 hours.

as the crow flies[1] : à vol d'oiseau.

<u>Exercice 3 :</u>

Is the following statement true or false ? Justify your answer.

« *If a sequence increases and l is an upper bound[2], then it converges to l* » .

upper bound[2] : majorant

Subject n°23

Sequences

Please, do not write on this document and do not forget to hand it back to the jury at the end of the test.

The treaty of Versailles was signed on 28 June 1919. The treaty required Germany to pay reparations for damages caused by World War I. Germany couldn't fulfill reparation payments, so weak was its economy, and a currency crisis ensued .

In January 1923, the US $ was worth 17,972 DM[1], and in July 1923, the US $ was worth 354,412 DM.

1)What was the increase of the exchange rate between the US $ and the DM over this six months period ?

2)What is this increase rate on a monthly basis ? Give the result in percentage, to 2 d.p.

3)a)Considering this monthly increase goes on, model this situation by a sequence and give its general formula.

b)Under these conditions, what is the predictable value of an US $ in November the same year?

c)In reality, the US $ was worth 4,200,000,000,000 DM in November 1923, can you comment on that ?

DM[1] : Deutschmark

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Subject n°24

Probabilities

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The three exercises are independent.

Exercise 1 :

In France, 2.5% of births result from in vitro fertilization (IVF). The probability of giving birth to twins is one fourth in that case, when it is one out of eighty otherwise.
As they walk by the street, two friends notice a pram[1] with twins; one of them says :
« *It's probably the consequence of an IVF* » .

What do you think about this comment ?

Exercise 2 :

In a small restaurant, 80% of customers have coffee and 40% have a dessert. ¾ of those who have a dessert also have coffee.

1. By means of a Venn diagram, represent the situation.

2. A customer of the restaurant is chosen at random . Work out the following probabilities.

   a. The probability that he had a dessert and coffee .
   b. The probability that he had neither dessert nor coffee.
   c. The probability that he didn't have dessert given that he had coffee.
   d.The probability that he didn't have dessert given that he didn't have coffee.

Exercise 3 :

Is the following statement true or false ? Justify your answer.

« *If two events are exclusive, then they are independent* » .

pram[1]: landau

Subject n°25

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PROBABILITY

During sport competitions some anti-doping tests are organized. Athletes can be declared positive (because they do drugs but also sometimes they are positive even if they don't use drugs) or negative (because they don't use drugs but also sometimes they are negative even if they do drugs).
The study is made on a group of 50 athletes.
Let's call n the number of athletes who do drugs in this group.
We know that:

- 95% of athletes who do drugs are declared positive;
- 10% of athletes who don't do drugs are declared positive.

1. Build a table showing the number (in terms of n) of athletes of each category.
2. Express, in terms of n, the number of mistakes made by the anti-doping commission.
3. We choose an athlete who has been controlled already.
    a. Show that the probability that he has been using drugs knowing that he has been declared positive is equal to : $\frac{0.95n}{5+0.85n}$
    b. Solve the inequation : $\frac{0.95n}{5+0.85n} > 0.95$.
    c. Give an interpretation of this result.

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<u>Subject n°26</u>

FUNCTIONS

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Naïm would like to watch some video-on-demand. He is comparing 3 offers from a provider :

- Offer #1 : A monthly fee of €13.99 for an unlimited number of videos.
- Offer #2 : A monthly fee of €5 plus €0.30 per watched video.
- Offer #3 : No monthly fee but €0.70 per watched video.

Reminder : “A monthly fee” is a fixed sum of money you have to pay each month just to access the service.

What offer Naïm should choose?

Subject n°27

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SEQUENCES

In 2019, the population of codfish offshore a coastline was estimated to 5 000 tons. Because of the overfishing, this population has significantly reduced close to this coastline.
In order to prevent the total extinction of this species, local authorities have decided to control and limit the fishing of codfish.

We suppose that, without the fishing factor, the population of codfish would stay constant at 5 000 tons.
In 2019, the quota of fishing for codfish under this coastline was fixed at 600 tons, Let's note $U(0) = 600$.
Local authorities have decided to drop this quota by 30 tons each year.

a) Calculate the quota of codfish, expressed in tons, that can be fished in 2020, called $U(1)$.
b) Calculate the quota of codfish, expressed in tons, that can be fished in 2021, called $U(2)$.
c) In a general way, we call $U(n)$ the quota of codfish, expressed in tons, that can be fished during the year (2019 + n).
   How do we call a sequence such as $U(n)$? Give its common difference.
d) Compute $U(10)$ and give a interpretation of this result in the context presented above.
e) Using your calculator, give the total quantity of codfish fished between 2019 and 2029 included.
f) Is this new regulation efficient enough to prevent the total extinction of codfish in this area?

(codfish = cabillaud)

Reminder : for any integer $n \geq 1$: $\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

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<u>Subject n°28</u>

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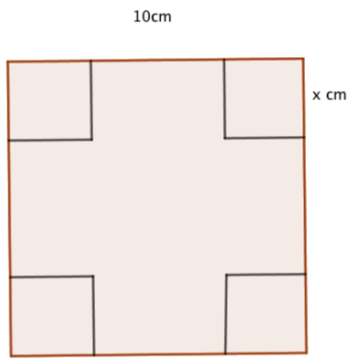
FUNCTIONS

The side of a square cardboard is 10cm long.
Zoe decides to cut a little square in each corner to fold its sides in order to obtain a jewelry box. The side of the little square is called x.



1. What are the possible values for x?
2. Determine the volume $V(x)$ of the jewelry box.
3. Let's call $V'(x)$ the derivative of $V(x)$.
   Calculate $V'(x)$ and check that $V'(5) = 0$.
4. Study the sign of $V'(x)$.
5. Determine the variations of V.
6. Zoe would like her jewelry box to be as big as possible. What value of x should she choose?

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Sujet n°33

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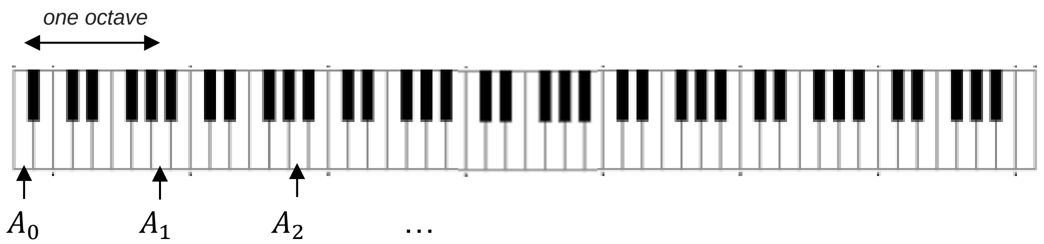
Sequences

In music, the “pitch” of a note is related to its frequency: the higher the frequency is, the higher the note is.
By multiplying the frequency of a note by 2, we obtain the same note, one octave above.
On a piano keyboard, the note of the first key is a A, called A_0. An octave higher is a second A, called A_1 and so on, as shown in the picture below.



The note of a tuning fork, whose frequency is 440 Hz, corresponds to A_4.
For any whole number n, let F_n be the frequency of A_n, in Hz, so that $F_4 = 440$.

a) Compute F_3.
b) What is the frequency of the first A on the piano keyboard?
c) What kind of sequence can we use to model (F_n)? Give its characteristics.
d) For any whole number n, express F_n in terms of *n*.
e) The human ear cannot hear a sound whose frequency is higher than 20,000 Hz. What is the higher A a person can hear?

NB: “the pitch” : la hauteur
“a tuning fork”: un diapason

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Sujet n°34

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Statistics

The table below is an extract from the monthly report on the labour market in the UK, issued by the UK Office for National Statistics in February 2020.

	Usual weekly hours of work	number of workers (in thousands)
Employees	from 0 up to 6 hours	278
	from 6 up to 15 hours	1,580
	from 16 up to 30 hours	5,196
	from 31 up to 45 hours	15,945
	from 45 up to 55 hours	4,727
Self-employed	from 0 up to 6 hours	147
	from 6 up to 15 hours	448
	from 16 up to 30 hours	1,136
	from 31 up to 45 hours	1,999
	from 45 up to 55 hours	1,297

1) According to this table, how many workers are there in the UK?
2) How is this labour force divided between employees and self-employed workers?
3) Knowing that the UK population is approximately 66.5 million inhabitants, what is the percentage of the working population in this country?
4) What is the median class of this series of data?
5) Compute the mean of usual weekly hours of work:
   a) for employees,
   b) for self-employed workers,
   c) for all workers.

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Sujet n°35

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Percentages and Sequences

1) 42 million alligator eggs are laid every year. Of those, only half hatch*.
Of those that hatch, three quarters are eaten by predators in the first month.
Of the rest, only 5% will grow into adults.

a) What percentage of alligator eggs will become adult alligators?
b) How many new adult alligators will there be every year?
c) Let's assume that:
   - there are as many males as females,
   - a quarter of females are fertile,
   - a fertile female lays an average of 35 eggs per year.

   What is the size of the alligator population when 42 million eggs are laid in a year?

2) In one of the numerous alligator farms in Florida, Happy Alligator Farm, taking into account births, deaths and sales, the total population increases by 4% a year.
There were 1,000 alligators in 2018.
For any whole number n, let P_n be the population of alligators in year $2018 + n$, so that $P_0 = 1{,}000$.

a) Compute P_1.
b) How many alligators will there be in this farm in 2020?
c) What kind of sequence can we use to model (P_n)? Give its characteristics.
d) How many alligators will there be in this farm in 2050?

To hatch: éclore

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Sujet n°36

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Functions

“Capitals” is a company which manufactures pens.
Its cost of production, in thousands of pounds, is modelled by the function C, defined and differentiable in the interval [0; 60]. C is a function of x, the number of thousands of pens produced, such as $C(x) = \frac{1}{4}(x^2 - 50x + 500)$

1) What does $C(0)$ represent?

2) Let’s assume that a pen is sold £2.50 and that all the pens are sold.
Show that the profit of the company, in thousands of pounds, can be modelled by the function
$$P(x) = -\frac{1}{4}x^2 + 15x - 125$$
x being the number of pens produced, in thousands.

3) Draw and comment the graph of function P. Use it to find out the approximate number of pens the company has to sell to be profitable.

4) Check your previous answer by computing the exact number of pens for which the company is profitable. What is the maximum profit it can make?

Subject n°37

FUNCTIONS

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Question 1 :

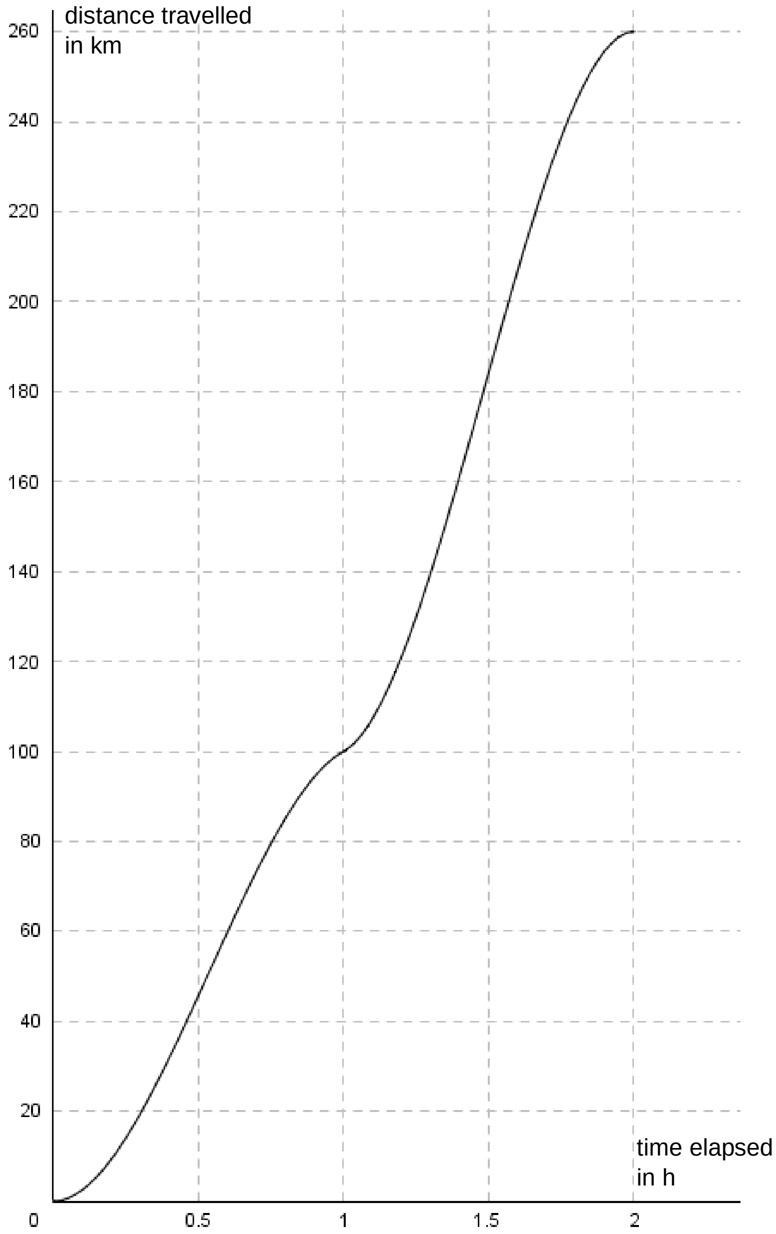
M. A. Driver and his wife went for a trip to France.

He drove during two hours on a motorway, and was very confident he had not exceeded the maximum authorised speed (130km/h).

But his wife was not so sure, and claimed that at some point they must have driven too fast.

You can see on the right the graph representing the distance they travelled in terms of the time elapsed.

Explain both M. and Mrs Driver's points of view, and find out who is right.



Question 2:

In 2010 M. T. Wizz created an internet company.

At first he worked alone, but then he started to hire people.

He estimates that when his business employs x workers, each term it earns a profit P(x) euros given by:

$P(x) = -11x^3 + 214x^2 - 384x$

1) When he started the business on his own, did he make any profit?

2) How many workers should be hired to maximise the profit of the business?

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Subject n°38

PROBABILITIES

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A teacher uses whiteboard markers in four colours: blue, red, green, and black.

This morning she has 12 markers in her case, among them 3 blue, 2 green, 3 red.

Some of them are new, and the others slightly used.

The proportion of new markers is: one out of two for the green, one out of three for the blue, two out of three for the red, and one out of two for the black.

<u>Question 1:</u>

Let us consider the following experiment: the teacher takes a marker from her case at random.

Then she checks the colour of the marker and whether it is a new one or not.

1) Choose notations and draw a probability tree to represent the outcomes of that experiment.

2) What is the probability that...

   a) ... the marker is red and new?

   b) ... the marker is new?

3) Knowing that the marker is new, what is the probability that it is black?

<u>Question 2:</u>

The problem with these markers is that when they are slightly old, they do not write long before fading.

Over the years the teacher has gathered some data about the markers: a new marker will write about fifty words with a sharp colour before fading, whereas a slightly old marker will write about ten words with a sharp colour before fading.

Over the whole year, each time she takes a marker at random from her case, what is the average number of words she can expect to be able to write before the ink fades?

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Subject n°39

SEQUENCES

Please do not write on this document, and hand it back to the jury at the end.

In a school, on the 1st of January, started an outbreak of flu.

This first day just one student was ill, then on the January 2nd there were four new cases, then on January 3rd six new cases, on January 4th seven new cases.

The head of school decided to model that epidemic, to see what would happen.

She created three sequences, as follows:

- (u_n) is the sequence of that gives the number of new cases in terms of the date
- (d_n) is the sequence such that for any integer $n \geq 2$: $d_n = u_n - u_{n-1}$
- (Δ_n) is the sequence such that for any integer $n \geq 3$: $\Delta_n = d_n - d_{n-1}$

1) Copy and fill the following table:

n	u_n	d_n	Δ_n
1	1		
2	4		
3	6		
4	7		

2-a) You should notice something about (Δ_n). What is it?

2-b) Assuming that what you noticed about (Δ_n) holds for any integer $n \geq 3$, add new lines to your table, and fill it in as long as u_n stays strictly positive.

2-c) How many students will have suffered from this flu from the 1st of January to the end of the epidemic?

3) The explicit formula for (u_n) is $u_n = a \times n^2 + b \times n + c$ where $a;\ b$ and c are parameters we are going to determine.

a) Show that for any integer $n \geq 2$: $d_n = a(2n - 1) + b$

b) For any integer $n \geq 3$: express Δ_n in terms of n

c) Then using the numbers in your table, determine $a;\ b$ and c.

Subject n°40

STATISTICS

Please do not write on this document, and hand it back to the jury at the end.

Question 1 : percentages

Here are the performances of two running clubs over the past two years:

The Rowdy Runners club has 120 runners, 40 of them female.

- 60% of the female runners have got a medal;
- 65% of the male runners have got a medal.

The Power Runners club has 200 runners, 140 of them female.

- 90% of the female runners have got a medal;
- 95% of the male runners have got a medal.

Two sports journalists discuss these results:
the first one declares that evidence shows male runners are more performant; but the second one hesitates and replies that he is not so sure.
What do you think? (justify carefully)

Question 2: averages

To build a wood deck in front of my living room I bought special deck screws.
I needed 50 of them, and so I thought I was very lucky when I found a packet of 50 deck screws in a DIY store nearby.
But I ended up very frustrated because actually there were only 48 screws in the packet, and so I couldn't finish my deck!
I then sent a complaint to the manufacturer, who answered that on average their packets did contain 50 screws. So the fact that my packet was two screws short was just bad luck, but not something they would take responsibility for.
Infuriated I got a customer's association to ask for a quality control. Thus 100 packets of screws were tested, and here is the distribution we got:

number of screws	frequency
47	18
48	18
49	19
50	20
52	7
54	4
56	5
58	9
total	100

Find the mean, mode and median number of screws per packet.

Then comment on the manufacturer's claim.

Nota: deck : (ici) terrasse
screws : vis
DIY : bricolage

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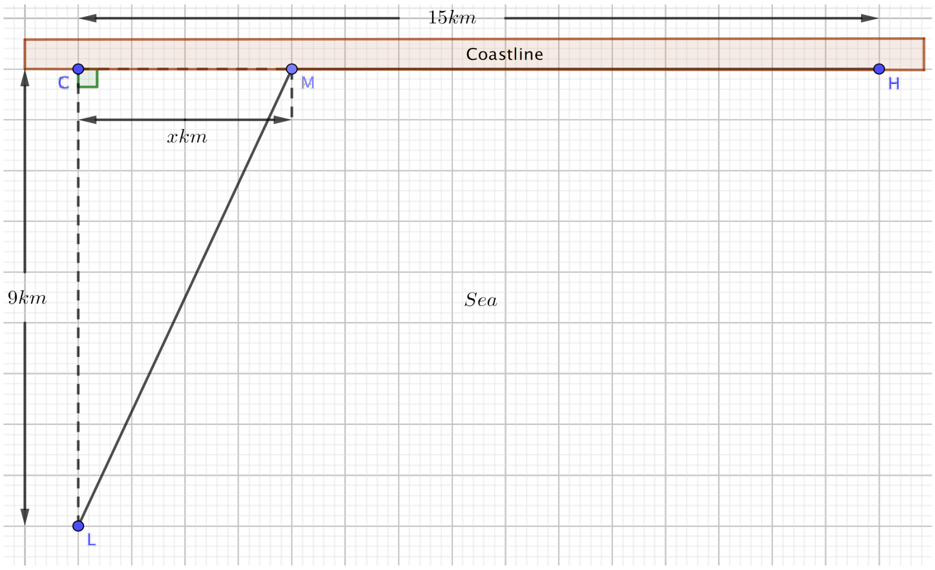
Subject n°45 : Functions

Please do not write on this document, and do not forget to hand it back to the jury at the end.

For some obscure reason, the lighthouse keeper *(gardien de phare) (point L)* must go to his house (point *H*), situated on the coastline *(CH)*.



He goes by boat at 4 km.h^{-1}, and walks on the shore at 5 km.h^{-1}.

(**Note :** the coastline *(CH)* is straight and the drift *(la dérive)* due to the current is insignificant).

The **goal** of the following steps is to answer the final question : where must he berth *(accoster)* on the coastline (point *M*) in order to **minimise** his travel time ?

1) Let x be equal to the length *CM*, with $0 < x < 15$. Prove that the **travel time** is given by the function f :

$$f(x)=\frac{1}{4}\sqrt{(x^2+81)}+\frac{1}{5}(15-x).$$

2) Study the variations of the function f on the interval [0 ; 15].

(Indication : prove that $f'(x)$ *has the same sign as :* $9(x-12)(x+12)$ *).*

3) Conclude on the distance x which **minimises the travel time**, and calculate it in hours and minutes.

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Subject n°46 : Percentages

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Success rates at the Bac exam : which is the better year ?

According to an enquiry *(une enquête)* in a class of Terminale :

	2018 Present	2018 Succeeded	2019 Present	2019 Succeeded
Non repeating	22	12	15	8
Repeating	3	3	10	9
Total				

(A repeating student : *un élève redoublant*)

The headmaster *(le proviseur)* said : “The year 2019 is clearly better than 2018 regarding the success rate at the Bac exam. I congratulate the teachers of this class”.

The delegate of students replied : “Wether a student was repeating his class or not, in 2019, the success rate was worse than in 2018. I don't congratulate the teachers”.

Who is right ? (The following questions will enable you to answer this question).

1º) Calculate the **global success rates** (in percentages) for the two years (considering the total numbers of students each year).

2º) Then calculate the success rates (in percentages) **in each category** : repeating students and non repeating students.

3º) Answer the initial question (who is right ?).

4º) What is your personal opinion about Mark Twain's thought :

"**Lies**, damned **lies**, and **statistics**"

(Mark Twain was a famous american writer, who liked making jokes).

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Subject n°47 : Probabilities

Please do not write on this document, and do not forget to hand it back to the jury at the end.

Galileo's problem.

The prince of Tuscany (Italy) asked one day to Galileo :

« When we toss a dice three times and add the results, why do we more often obtain the sum 10 than the sum 9, although those two sums can be obtained in six different ways ? »

1º) Find out the six different ways to express 10 and 9 as a sum of three numbers.

2º) Knowing that an elementary event of this random experiment is an **ordered** triplet

(a ; b ; c), explain Galileo's answer :

> « The event « The sum is 9 » is formed by 25 favorable outcomes, whereas
>
> the event « The sum is 10 » is formed by 27 favorable outcomes ».

3º) Calculate the probabilities of these two events and convert them in percentages.

4º) Knowing that the first throw gave 3, what is the probability that the sum of the three results gives 9 ?

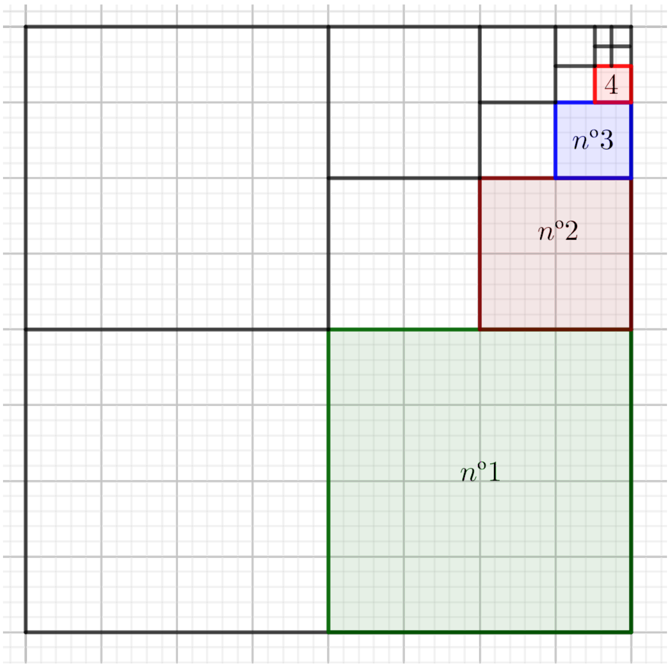
5º) Knowing that the first throw gave 3, what is the probability that the sum of the three results gives 10 ?

Subject n°48 : Sequences

Please do not write on this document, and do not forget to hand it back to the jury at the end.

Starting from a square of edge 5 cm, we build a first square (n° 1), then a second square (n° 2), etc… according to the following drawing.

For all $n > 0$, we call A_n the area of the n^{th} square (in cm²).



1°) Find out the **recurrence relation** of the sequence (A_n).

*(Indication : Express A_1 as a **fraction of cm²**, then express A_{n+1} in terms of A_n).*

What is the **nature** of the sequence (A_n) ?

2°) What is the **general term** A_n (in terms of n) ?

3°) Let $S_n = A_1 + A_2 + \ldots + A_n$.

Prove that $S_n = \frac{25}{3}\left(1 - \frac{1}{4^{n-1}}\right)$.

4°) What is the limit of S_n ?

Give a geometrical interpretation of this result.

Can this limit really be reached in the physical world ?

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Subject n°49

PROBABILITY

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Over 2 billion people speak English, making it the largest language by number of speakers, and the third largest language by number of native speakers. With 300 million native speakers, the United States of America is the largest English-speaking country whereas there are 67.5 million native speakers in the United Kingdom.

Source : https://en.wikipedia.org/wiki/English-speaking_world
Woman writing a letter, with her maid, by Johannes Vermeer

1.
   a. Compute the probability of picking randomly an Englishman in a population made out of these two nationalities. Round to the nearest percent.
   b. If we randomly pick 20 people, what is the probability that we find 5 English?

2. English and American spellings are *rigour* and *rigor*, respectively. A man is staying in a hotel and writes postcards with this word. A letter is taken at random from this spelling.

   a) Draw a tree diagram of this situation and compute the probability that this letter is a vowel.
   b) The letter taken is found to be a vowel. What is the probability that the writer is an Englishman?

Subject n°50

LINEAR EQUATIONS

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Mr Smith who is florist buys 302 flowers to make corsages* for the Winter Dance.

A carnation corsage used three flowers and a rose corsage uses two flowers.

A carnation corsage sells for $11, while a rose corsage sells for $20.

Rudolf Ernst
The Flower Vendor
Source : Wikipedia

1. The florist wants to optimize his production. Write simultaneous equations involving the number c of carnation corsages and the number r of rose corsages he should use.

2. How many flowers of each type should the florist use in order to maximize gross sales?

3. What amount will he get from the total sale?

4. He then sold all the carnation corsages and 20 rose corsages. What discount should he allow on the remaining rose corsages to sell the rest for $210 ?

* corsage : *petit bouquet*

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Subject n°51

SEQUENCES

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In the year 2010 a shop sold 150 smartphones.

Then each year the shop sold 10 more smartphones than the year before.

1. What type of sequence can we use to model this situation ?
2. Calculate the number of smartphones that the shop sold in 2020.
3. Calculate the total number of smartphones the shop will have sold from 2010 to 2020.

Source :commons.wikimedia.org

In the year 2010, the selling price of each smartphone was £100.

Then the selling price increased by 4.2% every year.

1. What type of sequence can we use to model this situation ?
2. Calculate the price of a smartphone today. Round the result to the unit.
3. When will the price reach £200 ?

Useful formula : The sum of the first n terms of such a sequence is $\frac{n(a_1+a_n)}{2}$ where n is the number of terms, a_1 is the first term and a_n is the last term.

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Sujet n°52

STATISTICS

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The ages of participants in a fund-raising walk were distributed as follows:

Age	Frequency	Cumulative frequency
$[10; 15[$	28	
$[15; 20[$	65	
$[20; 25[$	82	
$[25; 30[$	76	
$[30; 35[$	54	
$[35; 40[$	43	
$[40; 50[$	12	

a) Calculate an estimate of the mean age.

b) Complete the table.

c) Estimate the median, and the upper and lower quartiles.

d) Draw a box plot to represent these data.

e) What other statistics parameters could you use to describe these data ?

Annual Breast Cancer Walk in Swaziland

Source : Wikipedia

∽ Académie de Toulouse - Section Européenne ∾

∽ Session 2020 ∾

∽ Correction sujets 29 à 32 ∾

SUBJECT 29

1. The number of members of the country club is 350. Let x the number of women and y the number of men.
   We have two equations : $350 = x + y$ and $105 = (1/4)x + (1/3)y$.
   Hence, $x = (2/5)350 = 140$, $y = (3/5)350 = 210$ and $P(W) = 2/5$.
2. $P(T) = 3/10$.
3. $$P_T(W) = \frac{P_W(T)P(W)}{P(T)} = 1/3.$$
4. **a)** $X \in \{0, 1, 2, 3\}$
   b) X follows a binomial law with parameters $n = 3$ and $p = 0.3$.
   c) $P(X = 2) = 3 \times 0.3^2 \times 0.7$.

SUBJECT 30

EXERCISE 1

1. $$n(n+1)(n+2) = n + (n+1) + (n+2) \iff n^3 + 3n^2 - n - 3 = 0.$$
2. $$x^3 + 3x^2 - x - 3 = 0 \iff x = 1 \ or \ x = -1 \ or \ x = -3.$$
3. $n = 1$ is the only acceptable solution, so $(1,2,3)$ are the only triplet satisfying the condition.

EXERCISE 2

1. $A = 2(2 \times 499999)^2 + 1 = 2(2 \times 499999^2) + 1$ is odd.
2. $\forall n \in \mathbb{N}$, $B = 2n + 4 = 2(n+2)$, so Jane is right.

SUBJECT 31

EXERCISE 1

By induction, using

$$\left(x^{n+1}\right)' = \left(x \times x^n\right)'.$$

EXERCISE 2

1. $f'(t) = 0.003t^2 - 0.26t + 4$. f' has only one root in $[0;35]$, it is 20.
2. f is increasing on $[0;20]$ and f is decreasing on $[20;35]$
3. $t = 20$ and $f(20) = 41$.

SUBJECT 32

1. False : For example, $y = 2x + 1$ is an equation of a line.
2. True : $\overrightarrow{u} = (1;0)$ and point $A(0;y_0)$ (horizontal line).
3. True : $\overrightarrow{u} = (0;1)$ and point $B(x_0;0)$ (vertical line).
4. False : $2\overrightarrow{u}$ is also normal to d. Since d and d' are parallel, $2\overrightarrow{u}$ is normal to d'
5. False : The center of the circle is $\omega(1;-3)$ because

$$x^2 + y^2 - 2x + 6y + 6 = 0 \Leftrightarrow (x-1)^2 + (y+3)^2 = 4 \Leftrightarrow \omega M^2 = 4.$$

6. True :

$$x = 0 \Rightarrow (y+3)^2 - 3 = 0 \Rightarrow y = -3 \pm \sqrt{3},$$

and

$$y = 0 \Rightarrow (x-1)^2 = -5.$$

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SUBJECT 29

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PROBABILITIES

In Newbury (Berkshire), the local country club counts 350 members. The club manager has collected data about the tennis section of the club :

- one woman out of four joins the tennis section,
- one man out of three joins the tennis section,
- 30% of the members of the country club join the tennis section.

A member of the club is chosen randomly and the following events are definied :

- W : "The chosen member is a woman".
- T : "The chosen member joins the tennis section"

1. Prove that the probability of the event W is $2/5$.

2. Work out the probability of the event T.

3. Considering a member of the tennis section, what is the probability that this member is a woman?

4. In order to raise some funds for charity, the country club organizes a lottery every week during three weeks.

Every week, a member of the country club is chosen randomly and independently to take care of the lottery.

X is the total number of members of the tennis section chosen to take care of the lottery.

a) What are the possible values for X?

b) What is the law followed by X? Justify your answer.

c) Work out the probability of $X = 2$.

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SUBJECT 30

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NUMBERS AND EQUATIONS

Exercise 1

In this exercise, we want to find all the triplets of consecutive positive integers whose product is equal to their sum.

1. Show that solving this problem comes down to solving the following equation :

$$x^3 + 3x^2 - x - 3 = 0.$$

2. Show that the equation $x^3 + 3x^2 - x - 3 = 0$ is equivalent to :

$$(x-1)(x^2 + 4x + 3) = 0,$$

and then solve it.

3. Infer an answer to the initial question.

Exercise 2

1. Can you say if

$$A = 999998^2 + 1$$

is even or odd?

2. Jane says that, for all integer n, the following number

$$B = (n+1) - (n-1)$$

is even. Is she right?

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SUBJECT 31

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DERIVATIVES AND INDUCTION

Exercise 1

For this exercise, we asume that we **only** know :

— the derivative of the function $x \longmapsto x$,

— the derivative of the function $x \longmapsto \frac{1}{x}$

— the derivative of the product of two functions.

Show that for all real x and for all positive integer $n \geq 1$, the derivative of $x \longmapsto x^n$ is $x \longmapsto nx^{n-1}$.

Exercise 2

Using sensors[1] and a device called a "capnograph", a doctor measures the concentration of CO_2 in the air that is breathed out by a person over time (in seconds) during an effort. The unit of the concentration of CO_2 is the millimeter of mercury ($mm\ Hg$). The results of this stress test[2] are gathered in the following table :

t (s)	0	5	10	15	18	23	28	35
CO_2 ($mm\ Hg$)	5	21.875	33	39.125	40.712	40.397	37.032	28.625

It appears that the concentration of CO_2 can be modeled by the following function of t, in $mm\ Hg$:

$$f(t) = 0,001t^3 - 0,13t^2 + 4t + 5,$$

for $t \in [0;35]$.

1. Compute the derivative of f and study its sign.
2. Draw the table of variations of f.
3. For people in good shape, the maximum pressure of breathed out CO_2 is between 35 and 45 $mm\ Hg$. Will the doctor's diagnosis be optimistic?

1. capteurs
2. test d'effort

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SUBJECT 32

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FRAMED GEOMETRY

The plane is equiped with a frame $(O, \vec{i}, \vec{j})$.
Consider the following properties and decide wether they are right or wrong. Justify your answer.

1. $(\mathscr{P}_1)$: Every straight line has an equation of the form $y = ax$ (where a is a real number).
2. $(\mathscr{P}_2)$: Every equation of the form $y = y_0$ (where y_0 is a real number) is an equation of a straight line.
3. $(\mathscr{P}_3)$: Every equation of the form $x = x_0$ (where x_0 is a real number) is an equation of a straight line.
4. $(\mathscr{P}_4)$: If $\vec{u}$ is a normal vector to a line d, then there is a line d' which is parallel to d and such that $2\vec{u}$ is not normal to d'.
5. $(\mathscr{P}_5)$: $x^2 + y^2 - 2x + 6y + 6 = 0$ is the equation of the circle centered at $\Omega(1;3)$ and of radius 2.[1]
6. $(\mathscr{P}_6)$: The circle of $(\mathscr{P}_5)$ intersects the axes of the frame at two points.

1. The equation of the circle centered at $\Omega(a;b)$ and of radius r is :

$$(x-a)^2 + (y-b)^2 = r^2.$$

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Sujet n°41

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Probability

In the supermarket of your neighbourhood, you can use three different ways to check out. A statistic survey has shown that:

- 28% of the clients use the automatic check out way, reserved to the registered clients; a client using this way always finishes the payment in less than 1 minute;
- 52% of the clients use the credit card check out way; a client using this way has to pay with a credit card; among the clients using this way, 75% finish the payment in less than 1 minute;
- The other clients pay with cash.

A client is chosen at random. We consider the following events:

- A: “the client chooses the automatic check out way”;
- CC: “the client chooses the credit card check out way”;
- C: “the client chooses to pay with cash”;
- M: “the client takes less than one minute to finish the payment”.

We denote $\overline{M}$ the contrary event of event M.

1. **1.** Draw a probability tree corresponding to this situation. This tree will be filled in during the exercise.
2. **2.** Compute the probability $P(CC \cap M)$.
3. **3.** The statistic survey also showed that 70% of the clients finished the payment in less than 1 minute.
   - **a)** Justify that $P(C \cap M) = 0.03$.
   - **b)** Compute the probability that a client finishes the payment in less than 1 minute, knowing that he or she pays with cash.

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Sujet n°42

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Percentage and Functions

This is a true or false exercise. For each of the 4 statements, you need to say if it is either true or false and justify your answer.

1. The management of a company decides to decrease the budget allowed to the expenses for business travels.

   Statement 1: "Decreasing this budget by 6% every year during five consecutive years corresponds to decreasing the budget of 30% over this period of five years".

2. This enterprise produces and sells USB sticks. The monthly production varies between 0 and 10,000 sticks. The monthly benefit, expressed in thousands of euros, can be modelled by function *B* defined on the interval $[0; 10]$ by the expression:

   $$B(x) = -x^2 + 10x - 9,$$

   where *x* represents the number of thousands of USB sticks produced and sold.

   Statement 2: "When the enterprise produces and sells between 1000 and 9000 USB sticks, the benefit is positive".

   Statement 3: "When the enterprise produces and sells 5000 USB sticks, the benefit reaches its maximum".

   Statement 4: "When the enterprise produces and sells between 2000 and 8000 USB sticks, the average monthly benefit is equal to 78,000 euros".

   Reminder: The average of function f, defined and continuous on the interval [a; b], is: $\frac{1}{b-a} \times \int_a^b f(x)dx$.

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Sujet n°43

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SEQUENCES

In its issue of 1st of June 2019, a magazine proposes a sports plan to be fit for the summer holidays, which consists in walking a certain distance every day according to the following pattern:

On June 1 you have to walk 2000 meters.
Each consecutive day, you have to increase the distance covered the day before by 3% and add 150 more meters.

For any integer *n*, we define by u_n the distance you have to walk on the *n*th day of June.
Therefore $u_1 = 2000$ is the first term of this sequence.

1. Compute u_2 and u_3.
2. Justify that for any integer *n* we have: $u_{n+1} = 1.03 \times u_n + 150$.
3. For any integer *n*, we define $v_n = u_n + 5000$.
   Prove that sequence(v_n) is geometric with common ratio equal to 1.03 and give its first term.
4. Express v_n in terms of *n*.
5. Deduce that for any integer *n*, the expression of u_n in terms of *n* is:
   $$u_n = 7000 \times 1.03^n - 5000$$
6. Compute on which day the distance to be walked will exceed 4000 meters.
7. Compute the distance walked in total in the whole month of June.
   Reminder: for $n \geq 0$ *and* $q \neq 1$*:* $S_n = 1 + q + q^2 + \cdots + q^n = \frac{1-q^{n+1}}{1-q}$.

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Sujet n°44

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SEQUENCES

The technicians of an aquaculture farm want to set up the automatic dispenser of a chemical product to improve the quality of the water in a tank. The optimal concentration of the product, expressed in $mg.l^{-1}$ (milligrams per litre), is between 140 and 180 $mg.l^{-1}$.

At the beginning of the test, the concentration of the product in the tank is 160 $mg.l^{-1}$.

They estimate that the concentration decreases by 10% each week.

In order to follow the recommendations on the concentration of the product, they plan to program the dispenser to add a certain quantity of product each week.

They try to calculate it so that:

- The concentration of the product follows the recommendations without their intervention during 6 weeks;
- The quantity of consumed product is minimal.

They first try to set up the dispenser to add 10 $mg.l^{-1}$per week.

We observe the evolution of the concentration each week. This situation can be modelled by a sequence (C_n), where C_n represents the concentration of the product, in $mg.l^{-1}$, at the beginning of the *n*th week.

Hence $C_0 = 160$.

1. Justify that for any integer *n*, $C_{n+1} = 0.9 \times C_n + 10$.
2. We label (V_n) the sequence that for any integer *n* is given by: $V_n = C_n - 100$.
   - **a.** Prove that (V_n) is a geometric sequence with common ratio 0, 9 and first term $V_0 = 60$.
   - **b.** Express V_n in terms of *n*.
   - **c.** Deduce that for any integer *n*: $C_n = 0.9^n \times 60 + 100$.
3. Determine the limit of sequence (C_n) when n tends to infinity. Justify your answer.
4. Comment on the previous result according to the technicians' expectations.
5. After how many weeks will the concentration be lower than 140 $mg.l^{-1}$?
6. Does this set up correspond to the expectations?