Sujet n°1

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

FUNCTIONS

This exercise is a Multiple Choice Question exercise. For each of the four questions only one answer is correct. **For each question you need to justify your choice.**

The function f is given for any real number x in $]0; +\infty[$ by:

$$f(x) = \frac{e^{2x}}{x}$$

The second derivative of f is : $f''(x) = \frac{2e^{2x}(2x^2 - 2x + 1)}{x^3}$.

1. The first derivative of f is defined for x in $]0; +\infty[$ by:

a. $f'(x) = 2e^{2x}$

b. $f'(x) = \frac{e^{2x}(x-1)}{x^2}$

c. $f'(x) = \frac{e^{2x}(2x-1)}{x^2}$

d. $f'(x) = \frac{e^{2x}(1+2x)}{x^2}$

2. The function f:

a. is decreasing on $]0; +\infty[$

b. is a constant function on $]0; +\infty[$

c. reaches its minimum for $x = \frac{1}{2}$

d. reaches its maximum for $x = \frac{1}{2}$

3. The limit of function f when x tends to $+\infty$ is :

a. $+\infty$

b. 0

c. 1

d. e^{2x}

4. The function f :

a. is concave on $]0; +\infty[$

b. is convex on $]0; +\infty[$

c. is concave on $]0; \frac{1}{2}[$

d. is represented by a graph which admits a point of inflection

Sujet n°02

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Probabilities

In this exercise all the answers will be rounded to the nearest thousandth (10^{-3}).

A chemistry laboratory has just created a new doping test.
According to a survey, these are the results given by the test :
- If an athlete is doped, the probability that the test turns positive is 0.98. This is called the sensitivity of the test;
- If an athlete is not doped, the probability that the test turns negative is 0.995. This is called the specificity of the test.

During a track sports competition, an athlete is selected randomly and takes the test. We denote D the event "the athlete is doped" and T the event "the test is positive". We assume that the probability of the event D is 0.08.

1. Draw a probability tree representing this situation.

2. Prove that the probability for an athlete to test positive is 0.083.

3. Knowing that an athlete gets a positive test, compute the probability that he or she is doped.

4. The laboratory decides to commercialise the test if the probability of the event "an athlete with a positive test is doped" is bigger or equal to 0.95. Will the test be commercialised? Justify your answer.

---

track sports : athlétisme

Sujet n°3

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

SEQUENCES

At the end of 2020 an animal species counted 10,000 individuals. The evolution observed during the previous years leads to the hypothesis that this population will decrease by 5% each year.

In order to compensate for this decrease, there is a plan to reintroduce 200 individuals of this species at the end of each year, starting at the end of 2021.

1. How many individuals will there be in this species in 2026?
2. We denote (u_n) the sequence where u_n represents the number of individuals of the species during the year 2020 + n.
   Why can the sequence (u_n) be expressed by the formula : $u_{n+1} = 0.95 \times u_n + 200$?
   Which type of formula is it ?
3. For any integer n, we now consider the sequence (v_n) defined by: $v_n = 6000 \times 0.95^n$.
   Which type of sequence is (v_n)? Give its characteristics.
4. We assume that $v_n = u_n - 4000$.
   Give the expression of u_n in terms of n.
5. The spokesman of the NGO* in charge of surveying the evolution states that ''this species will not disappear, but unfortunately we won't be able to stop the loss of more than half of its population''.
   What do you think of this statement ? Justify your answer.

*Non governmental organization

Session 2024

Sujet n° 4

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Probability

90 people were asked which sport they liked the most. Their replies show that:

- 32 liked football (F),
- 34 liked cricket (C),
- 39 liked tennis (T),
- 12 liked football and tennis,
- 13 liked football and cricket,
- 11 liked cricket and tennis,
- 7 liked football, cricket and tennis.

1. Show that the probability that a person chosen at random likes only tennis is $\dfrac{23}{90}$

2. Show this information on a Venn Diagram. Justify your answer.

3. Find the probability that a person chosen at random likes exactly one of these sports.

4. Given that a person chosen at random liked tennis, work out the probability that this person also liked football.

Sujet n°05

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Statistics

Here are the names and heights of the players of the English team for the Women's Soccer World Cup 2023, in Australia and New Zealand :

| Name	Height	Name	Height	Name	Height
Mary EARPS	5' 8''	Esme MORGAN	5' 10''	Lauren JAMES	5' 9''
Hannah HAMPTON	5' 8''	Jessica CARTER	5' 6''	Rachel DALY	5' 6''
Ellie ROEBUCK	5' 8''	Keira WALSH	5' 6''	Lauren HEMP	5' 4''
Lucy BRONZE	5' 8''	Georgia STANWAY	5' 5''	Chloe KELLY	5' 6''
Niamh CHARLES	5' 8''	Ella TOONE	5' 4''	Beth ENGLAND	5' 5''
Alex GREENWOOD	5' 6''	Jordan NOBBS	5' 0''	Katie ROBINSON	5' 5''
Millie BRIGHT	5' 10''	Laura COOMBS	5' 5''	Alessia RUSSO	5' 9''
Lotte WUBBEN-MOY	5' 10''	Katie ZELEM	5' 4''		

1. What is the range of this data set?
2. In this team, what is the percentage of players measuring more than 5' 7''?
3. **a)** Compute the average height of these players (round your answer to the nearest inch).

   b) Knowing that 1' ≈ 30.48 cm and 1'' ≈ 2.54 cm, convert this mean in meters and centimeters.

4. **a)** Find the median, the lower and upper quartiles of this data set. Explain your method.

b) For their first match in the tournament, the English team faced the Haitian team whose statistics are given by the box-plot opposite: Compare the two teams.	
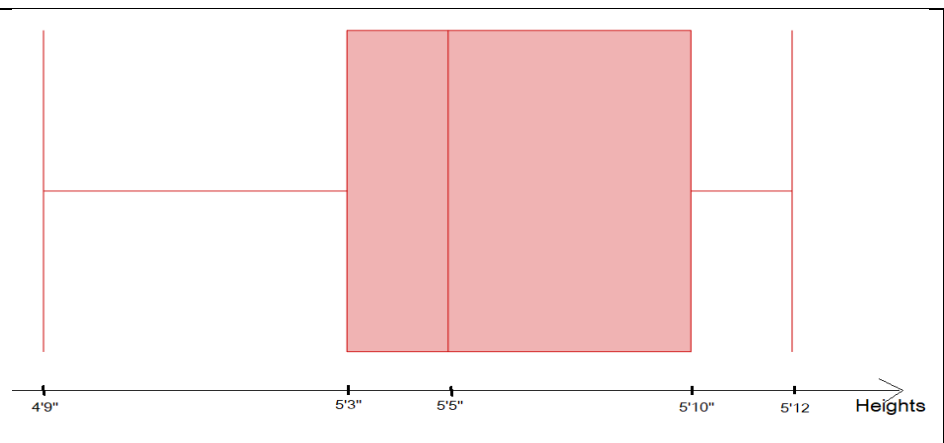
Sujet n°06

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Sequences

I.   400 athletes took part in the first Paralympic Games held in Rome in 1960.

In 1964, in Tokyo, they were 375. Since then, their number has increased by 285 athletes on average, every four years.

a) How many athletes took part in the Paralympic Games in Tel Aviv in 1968?

b) Let (p_n) be the number of athletes participating in the Paralympic Games of the year $1964 + 4n$. We know that $p_0 = 375$. Give the value of p_1.

c) For any whole number n, express p_{n+1} in terms of p_n. What is the nature of the (p_n) sequence?

d) For any whole number n, express p_n in terms of n.

e) Following that model, what would be the number of athletes taking part in the Paris Paralympic Games in 2024?

II.   **1)** For the Olympic Games held in Paris in 1900, we could count 22 women among the 997 competitors. Calculate the percentage of women. Interpret your result?

2) a) In 1948 in London, there were 390 women among the 4,104 competitors. Calculate the percentage of women.

b) Since then, the percentage of female athletes taking part in the Olympic Games has increased by about 10% every four years. Compute the percentage of women in the Olympic Games of Helsinki in 1952 and Melbourne in 1956 (round your result to the nearest hundreth).

3) a) Let (w_n) be the percentage of women participating in the Olympic Games of the year $1948 + 4n$. Give the value of w_0, w_1 and w_2.

b) For any whole number n, express w_{n+1} in term of w_n. What is the nature of the (w_n) sequence?

c) For any whole number n, express w_n in terms of n.

d) Following that model, what percentage of women can be expected in the Paris Olympic Games in 2024? Do you think it is realistic? Explain why.

Sujet n°07

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Statistics

1. When John started working, he was given a starting wage of £50 a week and was told it would increase by 4% every six months. How much will he make every week in 5 years?

2. A man invests £200 in a savings account at an annual interest rate of 7%. He makes no other deposit nor withdrawals. Interest is capitalized yearly and then it generates extra interest (ie: compound interest).

      a. How much money will the man have on his account at the end of the second year?

      b. After how many complete years will he have more than £500 on his account?

Sujet n°08

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THE NEW YORK MARATHON

David is getting ready for the New York Marathon (42.195km) which is to be held on the first Sunday in November.

He runs 10km every day in August. From September 1st onwards, he runs daily 0.8km more than the day before.

Let us call d_n the distance ran each day, with $n=0$ on August 31st, $n=1$ on September 1st, $n=2$ on September 2nd and so on.

1. Compute the first five terms of sequence (d_n)

2. Express d_{n+1} in terms of d_n

3. Calculate the distance run on September 30th

4. On what day does David run 42 km?

Sujet n°09

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A LINEAR EQUATION IN STANDARD FORM

On a forum, we can read:

"I'm very confused about the standard form of the linear equation Ax+By+C=0. Where do A, B, and C come from?

I'm supposed to translate point M(- 3;5) with a slope of 14 (the translation vector's coordinates are (1;14)), name M' its image point and find a linear equation on standard form for (MM')."

Could you help this person solve this problem:

- by using linear equations?

- by using vectors?

- by using any other method?

Sujet n°09

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Indiquer le thème du sujet

Sujet n° 10

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Sequences

The production of a piano factory (called the Treble Clef Ltd) follows an arithmetic sequence u, such as for every whole number n, u_n is the number of pianos manufactured during year $2000+n$.

In 2000, the factory produced 750 pianos, so $u_0 = 750$.

In 2005, the factory produced 1 200 pianos.

1. Give u_5 and justify the recurrence relation defining (u_n) ;

2. For any n , express u_n in terms of n.

3. In which year will the factory produce for the first time more than three times as many pianos as it initially did?

Another piano factory, Bass Clef Ltd, which was created in 2000, produced 350 pianos the first year. Since then, each year, the production of this company increases by 10%.

For any whole number n, let v_n be the number of pianos produced during year $2000+n$.

1. Compute v_0, v_1 and v_2.

2. What type of progression is (v_n) ?

3. For any counting number n , express v_n in terms of n.

4. Find out in which year Bass Clef Ltd will have produced at least as many pianos as Treble Clef Ltd.

Sujet n° 11

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Statistics

Two baseball coaches compare the number of runs scored by their teams in their last ten matches:

Rockets	0	10	1	9	11	0	8	5	6	7
Bullets	4	3	4	1	4	11	7	6	12	5

1. Show that each team has the same mean and range of runs scored.

2. Which team's performance do you suspect is more variable over the period?

3. Check your answer to question 2. by finding the standard deviation for each distribution.

4. Does the range or the standard deviation give a better indication of variability?

5. What is the median of runs scored for each team?

6. Which additional indication of variability can you calculate?

Sujet n° 12

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Probabilities

John likes soccer a lot but he doesn't have fun when he plays goal.

He has two coaches, Max and Diana. With Coach Max, the probability of being the Goalkeeper is 0.5 whereas with Coach Diana, the probability of being the Goalkeeper is 0.3.

Besides, Max is the Coach more often: 6 out of every 10 games.

We note M the event "Max is the coach" and G the event "John is goal".

1. Draw a tree diagram of the situation beginning with $P(M)$.

2. What is the probability that John will be the Goalkeeper?

3. What is the probability that John got Max as a Coach knowing that he plays goal?

4. Are M and G independent events?

Subject n°13

FUNCTIONS

***Please, do not write on the exam paper
and do not forget to give it back at the end of the test.***

A cylindrical tin must have a capacity V of 1 litre, so a volume of 1 dm³

We note r the radius of its base and h its height. (r and h are expressed in centimeters).

1). Show that the height h in terms of the radius r is $h = \dfrac{1000}{\pi r^2}$

Reminders : V = $\pi \times r^2 \times h$.

2) Prove that the total area of metal used to make the tin, in term of r is :

$$A(r) = 2\pi r^2 + \frac{2000}{r}$$

3) Justify that the value r_0 which minimises the area is $\sqrt[3]{\dfrac{1000}{2\pi}}$

Then give a value of r_0 rounded to the nearest hundredth .

4) Then compare the diameter and the height of the tin. (You can observe this for some tins in supermarkets).

Subject n°14

SEQUENCES

Please, do not write on the exam paper
and do not forget to give it back at the end of the test.

In most theatres, you have more seats in the rows towards the back and fewer seats in the rows up front as the room narrows towards the stage. Imagine having to clean up after a performance attended by children who were all given huge bags of popcorn. You need to plan the amount of time to do the job.

1. The theatre at a local civic centre has 13 seats in the first row, 15 in the next row, and the number of seats increases by 2 seats every row until there is a total of 30 rows in the theatre.
   a. How many seats are there in the last row ?
   b. How many seats are there in total in this theatre?

2. You start the job needing 40 seconds for the first seat. But the more you clean up, the quicker you get, so to finish one seat you need 2% less time than to finish the previous one.
   a. How many time do you need to clean up the 20th seat ?
   b. Calculate the total amount of time you will need to clean up the whole theatre. Round your answer to the nearest minute.

NB : $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ and, for q ≠ 1 : $1 + q + q^2 + \cdots + q^n = \frac{1-q^n}{1-q}$.

Subject n°15

PROBABILITIES

***Please, do not write on the exam paper
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An opinion poll made in a mountainous region about the construction of a dam gives the following results :

 • 65% of the interrogated persons are against the construction of the dam ;
• among the persons who are against the construction, 70% are ecologists ;
• among the persons favourable to the construction, 20% are ecologists.
We choose a person at random.

We note A the event « the interrogated person is against the construction » and $\bar{A}$ the contrary event.
We note E the event : « the interrogated person is an ecologist ».
We note F the event « the interrogated person is against the construction and is not an ecologist ».

1º) Draw a probability tree of the situation.

2º) Determine the probabilities P(A), $P_A(E)$ and $P_{\bar{A}}(E)$

3º) a) Calculate the probability that the interrogated person is against the construction of the dam and is an ecologist.
    b) Calculate the probability for an interrogated person to be in favour of the construction and to be an ecologist.
    c) Deduce from this the probability that the interrogated person is an ecologist.

4º) a) Prove that the probability of F is equal to 0.195.
    b) We choose at random five persons among those who have been interrogated during this poll. What is the probability that at least one of these persons is against the construction of the dam and is not an ecologist? (We suppose that the choice of these five persons is independant one from another).

NB : a poll: un sondage.
       a dam : un barrage
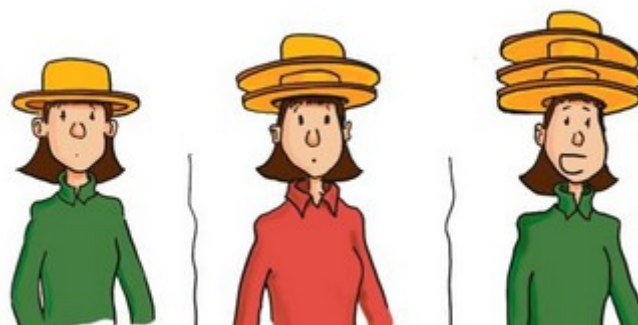
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Sujet n° 16

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Probability

To go to the university in the neighborhood of London, students cycle or go by public bus.
If they take their bicycle, they only have a quarter of chance of being late. If they go by bus, they have half a chance to be late.
If they are on time one day they will take the same means of transportation the following day, if they are late they will switch.
The probability that a student takes the bus on day 1 is 1/3
One student is randomly selected at the university.

1) Display this information in a tree diagram for day 1

2) What is the probability that the student will be late on day 1?

3) What is the probability that the student cycles on day 1, given that he is not late?

4) Explain how to find the probability that he takes the bus on day 2.

Session 2024

Sujet n° 17

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Sequence

1) Question : Guess my wearings the day after…

Monday   Tuesday   Wednesday

(from Apmep)

2) The first three terms of an arithmetic sequence (U_n) are given by
$U_0 = 2x + 5$ $U_1 = 5x + 1$ $U_2 = 6x + 7$
Show that x = 5 is the only value which gives an arithmetic sequence.

3) Let us consider a triangular wall with layers of bricks as shown.

If I have 171 bricks, how many layers could I build ?
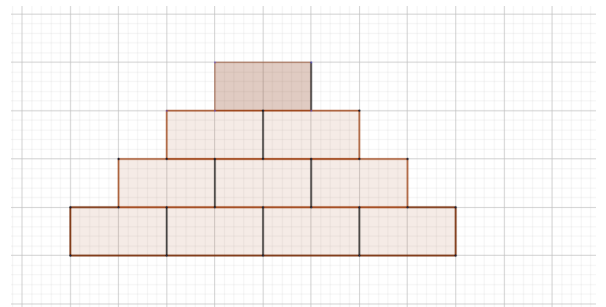
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Sujet n° 18

Functions

1) True or false : Justify your answer.

a) Affirmation :The graph of y = x² sin(x) meets the y-axis at exactly two points.

b) Knowing that x is a real number, different from 1, let us consider g(x) = $\dfrac{x^2-3x+2}{x-1}$

Affirmation : The graph of y = g(x) is a straight line

2) Let us consider f(x) = $\dfrac{2x+3}{x^2+4}$

a) Determine the domain of f

b) Determine the derivative function f'.

c) For x belonging to [0; 8] , the function f modelises the altitude during the flight of a bird.
x represents the time in minutes and f(x) the altitude in meters.
Knowing that the bird takes its soaring* from a branch, describe the path of the bird (the shape of the curve) during the 8 min.

soaring = envol

Sujet n°19

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Functions

A baseball player swings and hits a pop fly* straight up in the air to the catcher.

The height h of this ball (in meters) can be modelled by a quadratic function depending on time t measured in seconds after the hit.

The function can be written as $h(t) = -4.9\,t^2 + 17.4\,t + 1$, for each t in [0 ; 4].

1) How high was the ball when it was hit ?

2) Study the variations of the function h over interval [0 ; 4].

3) How long does it take the ball to reach its maximum height ? What is this maximum height ?

4) When does the ball strike the ground if the catcher does not catch it ?

* Pop fly = chandelle

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Probabilities

Sean's favorite season is winter.

When he goes out in winter, the chance he wears a woolly hat is 5 out of 8.

If he wears a hat, the chance that he wears also a scarf is 2 out of 3.

If he does not wear a hat, the chance he wears a scarf is 1 out of 6.

Let's label the events :

- "H", Sean wears a woolly hat.
- "S", Sean wears a scarf.

1) Complete the tree diagram.
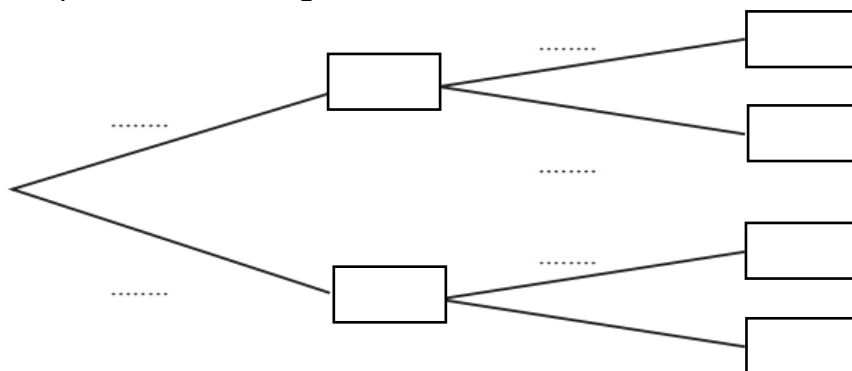
2) Compute the probability that

   a) Sean does not wear a hat nor a scarf.

   b) Sean wears a hat but does not wear a scarf.

   c) Sean wears a scarf.

3) Compute the probability Sean wears a hat given that he wears a scarf.

4) Knowing that when he does not wear a hat nor a scarf, the probability he does not wear gloves is 3 out of 10, compute the probability Sean doesn't wear any winter accessories (no gloves, no hat and no scarf).

Sujet n°21

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Statistics

The **Saffir–Simpson hurricane wind scale** (**SSHWS**) classifies hurricanes or tropical cyclones formed in the Atlantic Ocean into six categories distinguished by the intensities of their winds. This scale does not take into account other potentially deadly hazards such as storm surge, rainfall flooding, and tornadoes.

The highest wind speed averaged over a one-minute interval is measured at 10 m above the surface and gives the category (T is just for a storm and 1 to 5 are for hurricanes) :

Category	T	1	2	3	4	5
Wind speed in m/s	18 to 31	33 to 42	43 to 49	50 to 58	58 to 70	> 70
Wind speed in km/h	63 to 118	119 to 153	154 to 177	178 to 208	208 to 251	> 251

Adapted from Wikipedia and www.nhc.noaa.gov

In 2022, 14 storms were labelled from A to N from June the 1st to November the 30th.

In the table below are given the category and the maximum wind speed for these storms :

Name	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Category	T	T	T	1	2	4	T	T	5	1	T	1	1	1
Wind speed (km/h)	110	85	65	150	165	220	100	65	260	140	95	140	140	120

Argue to determine if the following statements are true or false.

Statement n°1 : On average in 2022, the maximum wind speed was around 132 km per hour.

Statement n°2 : At least 25% of these tropical storms were only storms and not hurricanes

Statement n°3 : Half of these storms got a maximum wind speed greater than 130 km/h

Statement n°4 : Comparing to 2005 (27 recorded storms), 2022 was a calm season

2005 box plot

Wind speed in km/h
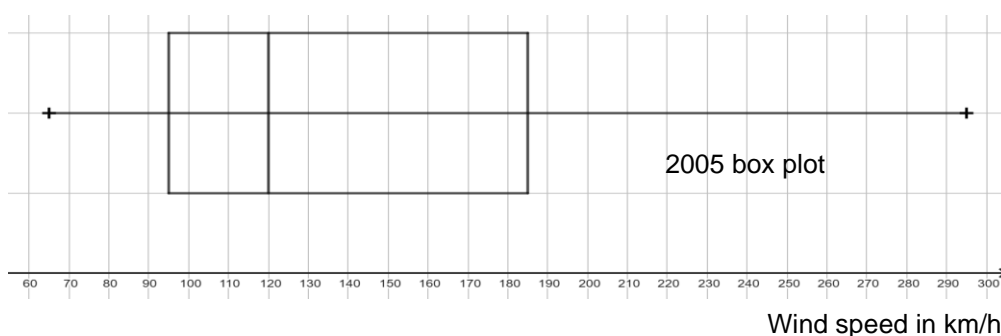
Sujet n°22

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Exponential functions

1. Identify whether the following statements represent an exponential function. Explain your answer.
   (a) The population of a pack of wolves increases each year by 25 individuals.
   (b) A population of bacteria decreases by a factor of $\frac{1}{8}$ every 24 hours.
   (c) The value of a coin collection has increased by 3.25% annually over the last 20 years.
   (d) For each training session, a personal trainer charges his clients 5£ less than the previous training session.
   (e) The height of a projectile at time t is represented by the function
   $$h(t) = -4.9t^2 + 18t + 40.$$

2. For the following questions, determine whether the table could represent a function that is linear, exponential, or neither.
   Justify your answers.

   (a)
x	1	2	3	4
$f(x)$	70	40	10	-20

   (b)
x	1	2	3	4
$m(x)$	80	61	142.9	25.61

   (c)
x	1	2	3	4
$g(x)$	3	6	12	24

   (d)
x	1	2	3	4
$h(x)$	10	20	40	80

Sujet n°23

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Quadratic equations and probabilities

Consider the quadratic equation

$$(E) \quad x^2 + px + q = 0.$$

The coefficients p and q are both among the following numbers : 1, 2, 3, 4, 5, 6.
p is given by the roll of a fair die.
q is given by a second roll of the same die.

1. What are the possible values for p and q?
2. Determine the probability of the following events :
   (a) " The equation (E) has two different real roots".
   (b) " The equation (E) has one real root".
   (c) " The equation (E) doesn't have any real root".

Sujet n°24

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Sequences

"The Scottish blackface sheep breed are one of the hardiest sheep breeds in Scotland. They are the backbone of the Scottish sheep industry. All the blackfaces are horned, with black or black and white face and legs. The females have a strong mothering ability to rear lambs in extreme terrain. The breed is ideal for large areas of hill country and is able to produce sheep for every climatic condition" *(scottish-blackface.co.uk)*

A Scottish blackface is commercialised after an eighteen months breeding.

When starting raising a flock of blackfaces, a breeder must take into account the market price and the profits he hopes in order to determine the size of the herd.

Since the sale happens eighteen months later, the market price will be different. So the supply of the sheep is made on the market depending on the price p_{n-1} whereas the demand is depending on the price p_n, eighteen months later.

We can simplify the situation with the following functions supply and demand :

$$S(n) = a + bp_{n-1} \ \ and \ \ D(n) = c + dp_n.$$

1. Say precisely what is p_n in that context.
2. What is the sign of b? Of d?

---

breed : élevage
flock/herd : troupeau

3. We study a case with the following values :

$$a = -10, \quad b = 1, \quad c = 5, \quad d = -2.$$

Consider that for every period of eighteen months, the supply and the demand are balanced.

(a) Show that, for all $n \in \mathbb{N}^\star$, $p_n = 7.5 - 0.5 p_{n-1}$.

(b) Consider the sequence (q_n) such that for all $n \in \mathbb{N}$,

$$q_n = p_n - 5.$$

Show that (q_n) is a geometric sequence.

(c) Give the expression of q_n in terms of n and, then, the expression of p_n in terms of n.

4. What can you say about the result?

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Sequences

9 is a square number, so you can arrange nine dots in a square shape:

This idea works for any kind of shape.

Below are the first four equilateral triangles you can get by arranging dots:

They define the triangular numbers.

1) Let us label (t_n) the sequence of triangular numbers: thus $t_1 = 1$, $t_2 = 3$

   a) Determine t_3, t_4, t_5

   b) If n is a whole number, what recurrence formula can you write between t_{n+1} and t_n?

2) Write t_3 , t_4, t_5 as a sum of numbers and determine the nth term of this sequence, for any whole number n.

3) Real-life applications of hexagonal numbers:

A suspended bridge requires a very strong cable.

One way to achieve that is to compact many steel strands together in a hexagonal shape.

The example on the right is a "size five" cable, it is made of 61 strands:

   a) Explain how you can use the diagram below to justify that there are 61 strands.

   b) Imagine a "size ten" cable.

   How many strands would you need?
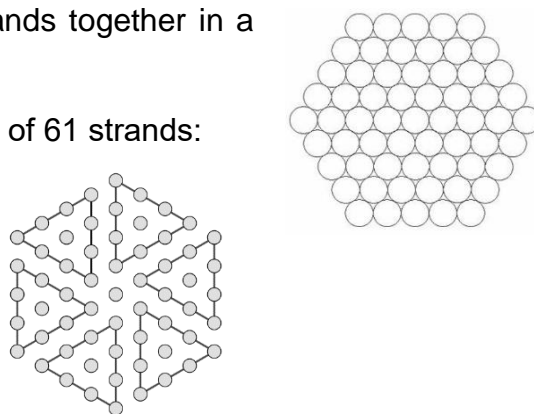
NB: here are two formulas you could want to use :

$$\sum_{i=1}^{n} i = \frac{(n+1)n}{2} \ , \ \sum_{i=0}^{n} q^i = \frac{1-q^{n+1}}{1-q} \ for \ any \ real \ number \ q \neq 1$$

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Functions

Part 1:

Here are expressions for functions:

$$f(x) = 2x - 3 \quad ; \quad g(x) = x^2 - 5x + 6 \quad ; \quad h(x) = \frac{1}{x} - 2 \quad ; \quad k(x) = -\frac{1}{x^2 - 1} + 3$$

Sketch the different graphs and explain them.

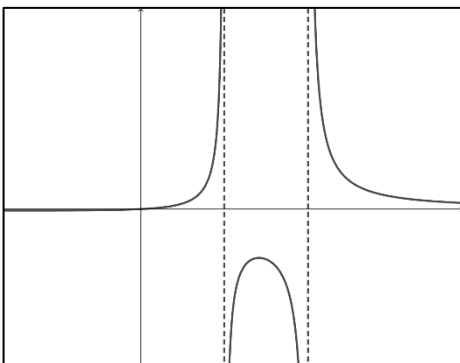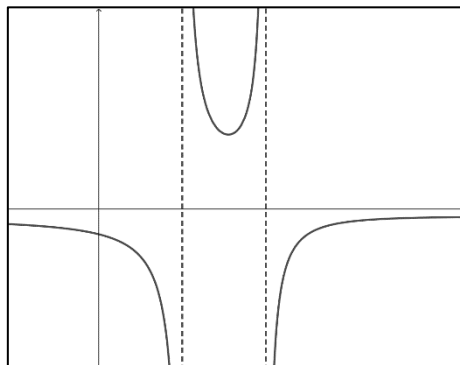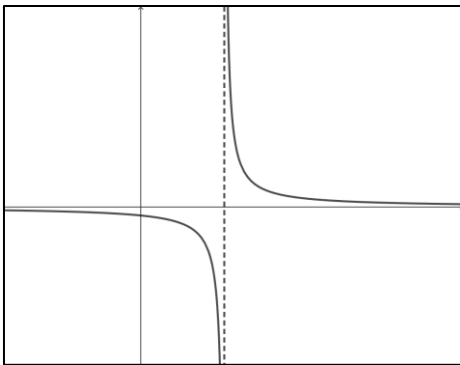Focus on what is important: no need of details.

Part 2:

Let us consider the expression $f(x) = \frac{x - 4}{x^2 - 3x + 2}$

Here are graphs:

none of them is exactly the graph of f, but one of them looks like it.

Choose the good one and explain it.
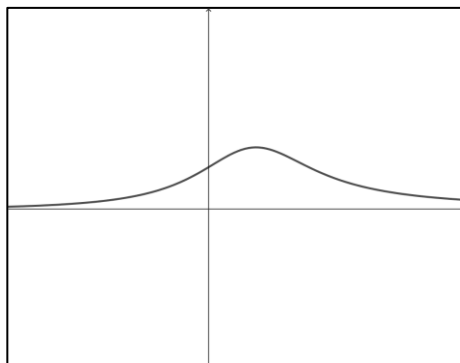
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Probabilities

Question 1:

In a bag are red and blue beads.

We know that there are 7 red beads.

We pick a first bead at random, then put it back in the bag, then again pick a bead at random.

Knowing that the probability that the two beads are red is $\frac{1}{16}$, work out the total number of beads in the bag before we removed anything.

Question 2:

In a bag are n beads, where n is a whole number.

4 of them are red, and the rest are blue.

We pick a bead at random, do not put it back in the bag, then pick another bead at random.

We know that the probability that we take two blue beads is $\frac{1}{3}$:

a) show that $n^2 - 13n + 30 = 0$

b) work out the value of n.

Optional question:

In a bag are red beads and blue beads.

The ratio of red beads to blue beads is 4:1.

We pick two beads at random.

Knowing that the probability that the two beads are blue is $\frac{1}{29}$, work out how many beads there were in the bag before we removed anything.

NB: "beads" means "perles".

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SEQUENCES

1. Here are the first terms of an arithmetic sequence:

$$3 \quad 8 \quad 13 \quad 18$$

   a. Write an expression, in terms of n, for the nth term of this sequence.

   b. Is 107 a term in this sequence? Explain your answer.

2. The nth term of an arithmetic sequence is $4n + 3$, where n is a whole number.

   a. Calculate the first four terms of this sequence.

   b. What is the common difference of this sequence?

   c. Determine whether 110 is a term in this arithmetic sequence.

   d. Find an expression for the sum of the nth term and the $(n-1)th$ term of this sequence.

   Give your answer in its simplest form.

3. Here are the first five terms of an arithmetic sequence:

$$3 \quad 7 \quad 11 \quad 15 \quad 19$$

   Prove that the difference between the squares of any two consecutive terms of the sequence is always a multiple of 8.

Sujet n°29

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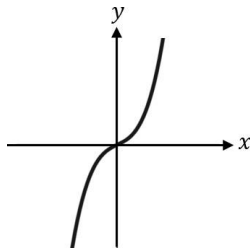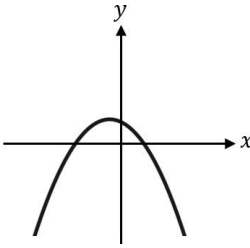

STATISTICS

Some pupils in a year group sit an exam at the end of the year. Their results are given in the table below.

Exam mark (%)	Frequency	Cumulative frequency
$0 < x \le 20$	3	
$20 < x \le 30$	10	
$30 < x \le 40$	12	
$40 < x \le 50$	24	
$50 < x \le 60$	42	
$60 < x \le 70$	16	
$70 < x \le 80$	9	
$80 < x \le 100$	4	

1. How many pupils sat at this exam?

2. Compute the cumulative frequencies.

3. To success, the mark has to be greater than 60%. How many pupils passed the exam?

4.
   a. Use the table to draw a cumulative frequency graph on the graph attached.

   b. Use the graph to estimate the median. What does it mean in that context?

   c. Use your graph to estimate the interquartile range. What does it mean in this situation?

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GRAPH : question 4a
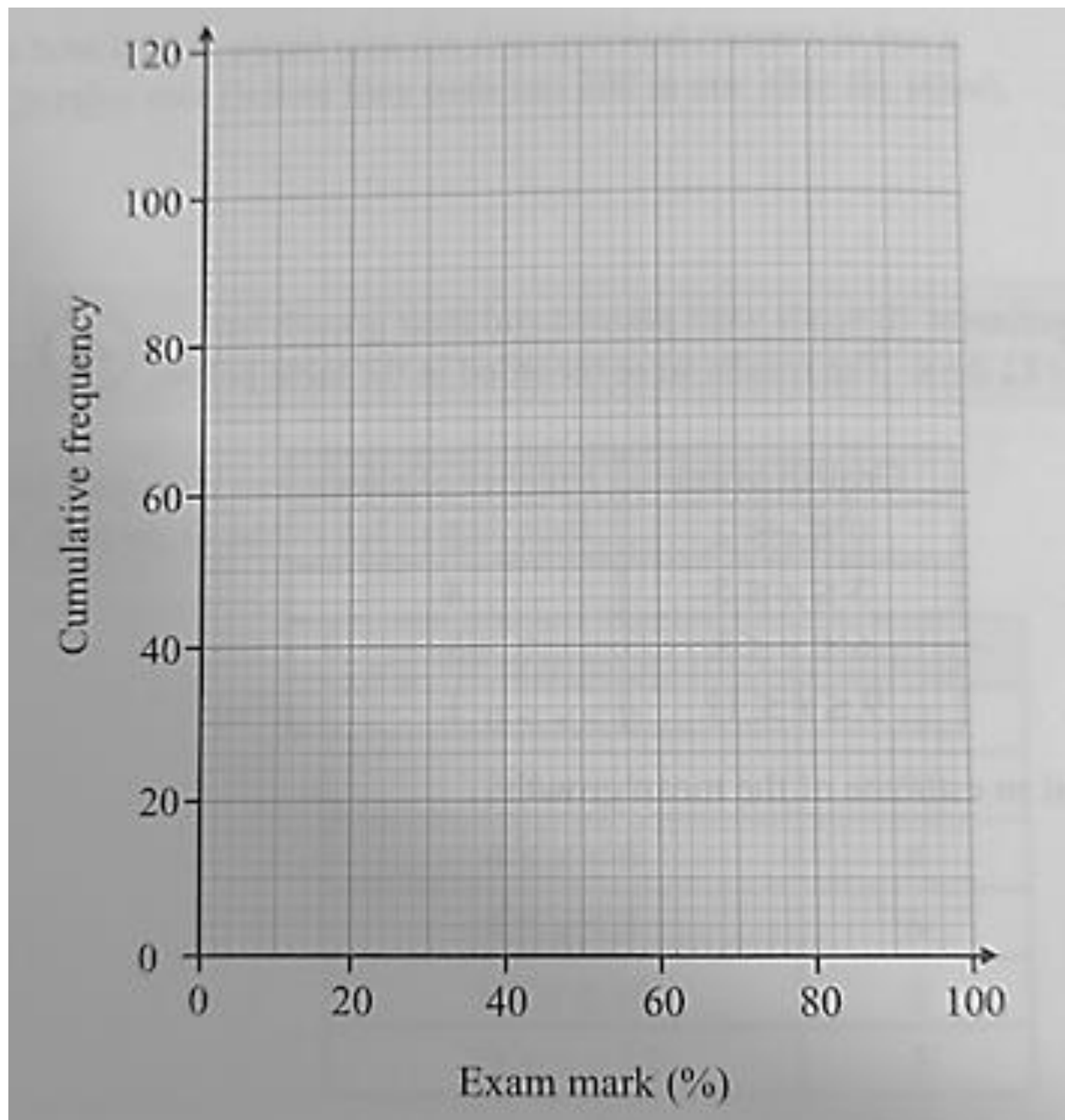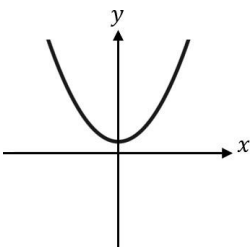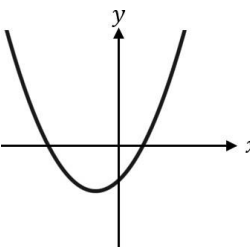
Sujet n°30
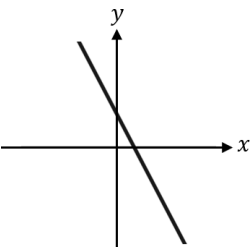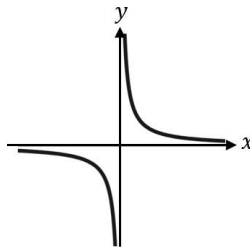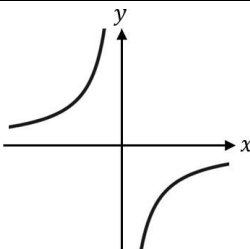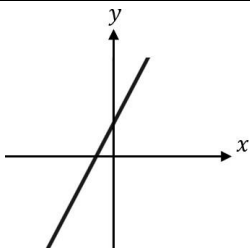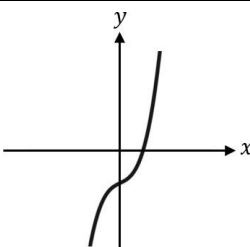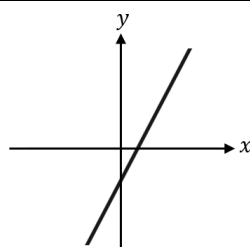
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FUNCTIONS

Please note : NO CALCULATOR ALLOWDED
Match up the sketch with the function. Explain your choices.

1		**6**		**A**	$y = x^2 + 2x - 3$
				B	$y = \dfrac{2}{x}$
2		**7**		**C**	$y = x^3 + x$
				D	$y = -\dfrac{10}{x}$
3		**8**		**E**	$y = 3 - 2x$
				F	$y = 2 - x - x^2$
4		**9**		**G**	$y = x^2 + 1$
				H	$y = 2x - 3$
5		**10**		**K**	$y = 2x^3 + x - 3$
				L	$y = 2x + 3$

1	2	3	4	5	6	7	8	9	10

(From Higher GCSE 2018)

PROBABILITY

Soccer Game

You are off to soccer, and love being the Goalkeeper, but that depends on who the Coach today is:

- with Coach Sam the probability of you being Goalkeeper is **0.5**
- with Coach Alex the probability of you being Goalkeeper is **0.3**

Sam is Coach more often ... about 6 out of every 10 games (a probability of **0,6**).

So, what is the probability you will be a Goalkeeper today?