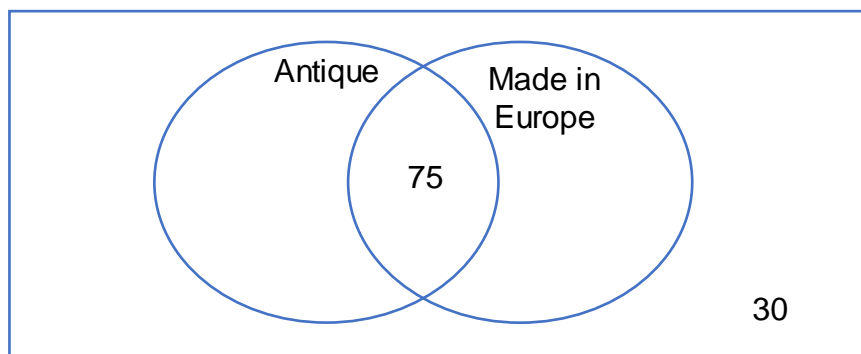## Sujet n°1

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Probability

A museum has a collection of 200 crockeries.

The two-way table and the Venn diagram show some information about the crockeries.

	Made in Europe	Made outside Europe	Total
Antique			
Not antique			
Total		80	200

1. Complete the table and the Venn diagram. Justify your results.

2. Find the probability that a randomly chosen piece of crockeries is made in Europe.

3. Find the probability that a randomly chosen piece of crockeries is antique and made in Europe.

4. Find the probability that a randomly chosen piece of crockeries is antique given that it is made in Europe.

Vocabulary : Crockery = Vaisselle

Sujet n°2

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Reasoning and arithmetic

1. **Proof**

   a. Prove that the difference between the squares of two consecutive even numbers is always a multiple of 4.

   b. Prove that the product of an odd number and an even number is even.

   c. Prove that the sum of any three odd numbers is odd.

2. **True or false?**

   "The difference between any two consecutive square numbers is always a prime number."

Sujet n°3

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Probabilities

500 new vehicles have been sold by a garage sale last year.

85 are ethanol vehicles.

13 % are electric vehicles.

7.8% are hybrid vehicles (both electric and ethanol).

1. Complete the following table :

	Ethanol	Not ethanol	Total
Electric			
Not electric			
Total			500

2. The type of a vehicle is chosen at random.

   a) What is the probability that this vehicle doesn't run on ethanol ?

   b) What is the probability that this vehicle runs on ethanol but not on electricity ?

   c) Knowing that the chosen vehicle runs on electricity, what is the probability that this vehicle also runs with ethanol ?

<u>Sujet n°4</u>

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

In order to employ someone, a company offers two different pays.

→ **Pay 1:** an initial monthly pay of £1,100 and a monthly increase of £150 per year.

→ **Pay 2:** an initial monthly pay of £1,100 and a monthly increase of 10% per year.

Question: Which pay is the best if the employee stays 10 years in the company?

Sujet n°5

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

An important industrial company discharges 50,000 tons of waste in 2019.

To respect new antipollution norms, it must reduce this quantity to less than 30,000 tons in 10 years.

The company undertakes to reduce the quantity of waste of 4 % per year.

1) If the company produces 48,000 tons of waste in 2020, is the undertaking respected ?
2) Let r_n be the number of tonnes discharged in year $2019 + n$.
   a. Show that (r_n) is a geometric sequence. What is its common ratio ?
   b. Express r_n in terms of n.
3) Compute the quantity of waste expected in year 2029. Round your answer to the nearest ton. Will the undertaking be respected in 2029 ?
4) Would an annual rate of decrease of 5 % allow to respect the undertaking ?

Vocabulary :

- undertake = prendre l'engagement
- discharge = rejeter

Sujet n°6

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Statistics

Part I :

In a sale, the marked prices are reduced by 30%.

a) Calculate the sale price of a jacket if the marked price is £350.
b) Find the marked price of a dress if the sale price is £168.

Part II :

The fraud department studied a sample made of 200 boxes supposed to contain 170 g of cheese. The table below gives the results :

Weight (g)	166.5	168	168.5	169	169.5	170	170.5	171	171.5	172
Frequency	1	6	12	21	36	48	34	18	14	10

a) Compute the mean (average) and the median of this data set.
b) Calculate the range and the interquartile range.
c) The regulations require a median of 170 g and an interquartile range of 1.35 g. Does this sample meet the requirements ?
d) The company changes the setting of weighing machines when more than 20 % of the boxes weigh 171 g or more. Should it do so ?

<u>Sujet n°7</u>

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

As he is 1.75 m tall and weighs 120 kg, Brian is too fat. So, on the advice of his doctor, he's been beginning a diet in order to slim, on January 1st, 2020. Every month he loses 5% of the weight he had the first of the month, but, on the last day of the month, he breaks down, eats too much, and he takes back two kg.

We denote p_n Brian's weight after n months of such a diet. So p_0 = 120 kg.

1) Calculate p_1 and p_2.
2) Justify that for any $n : p_{n+1} = 0.95 \times p_n + 2$.
3) Explain why (p_n) is neither a geometric sequence, neither an arithmetic sequence.
4) The Body Mass Index (BMI) is the ratio of the weight (in kg) by the square of the size (in meters). What is Brian's BMI when he begins his diets?
5) You're considered overweight when your BMI is more than 25. What should be the maximal weight for Brian in order not to be overweight anymore?
6) If he goes on with his diet, calculate when he may reach his goal.
7) Now we assume that for any $n : p_n = 80 \times 0.95^n \, 40$.
   Using this formula, try to find again the result of question 5.
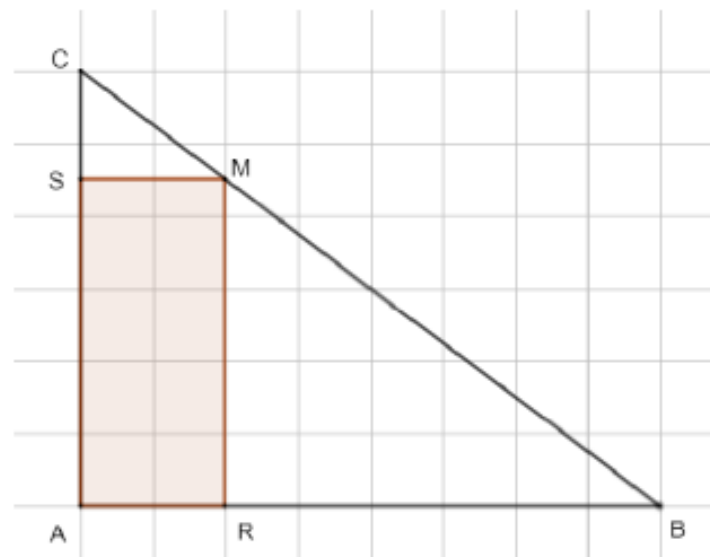8) What do you think of the Brian's weight tendency on a very long time?
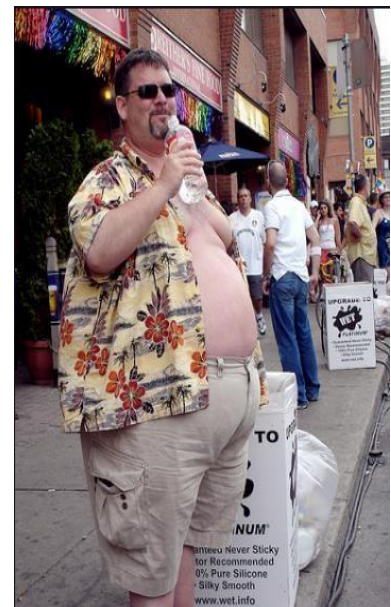
Sujet n°8

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Functions - optimization

We consider a right angle ABC triangle, such as AB = 8 cm and AC = 6 cm.

For every moving point on the [BC] line segment, we build the ARMS rectangle as shown on the geometric shape above.

Let x be equal to the length AR. Let x be the length of line segment AR in cm, and $f(x)$ the area of ARMS rectangle in cm².

1. Which interval does x belong to?
2. Make a conjecture about the variations of the f function.
3. Using Thales Theorem, prove that $f(x) = -\frac{3}{4}x^2 + 6x$.
4. Using your own calculator, try to estimate the maximal area of the ARMS rectangle.
5. Now, using your own math skills, prove your previous result.
6. For which values of x is the rectangle area greater or equal to 9?

Sujet n°9

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Probabilities

The following system of categorizing social class is widely used in Great Britain:
- Class A: Higher managerial, administrative and professional
- Class B: Intermediate managerial, administrative and professional
- Class C1: Supervisory, clerical and junior managerial, administrative and professional
- Class C2: Skilled manual workers
- Class D: Semi-skilled and unskilled manual workers
- Class E: State pensioners, casual and lowest grade workers, unemployed with state benefits only.

	Class A	Class B	Class C1	Class C2	Class D	Class E
Share in population	4%	23%	28%	20%	15%	10%

Source : https://en.wikipedia.org/wiki/NRS_social_grade

A survey institute conducted a survey to determine the influence of social class on the vote for Brexit, where voters only had to choose between Remain and Leave (the European Union).

	Classes A and B	Class C1	Class C2	Classes D and E
Vote for *Leave*	41%	48%	62%	64%

Source : https://www.ipsos.com/ipsos-mori/en-uk/how-britain-voted-2016-eu-referendum

We randomly choose a voter among all respondents in the survey. Let us consider the following events:
- L : "the respondent votes for Leave";
- AB : "the respondent belongs to class A or B";
- C_1 : "the respondent belongs to class C1";
- C_2 : "the respondent belongs to class C2";
- DE : "the respondent belongs to class D or E".
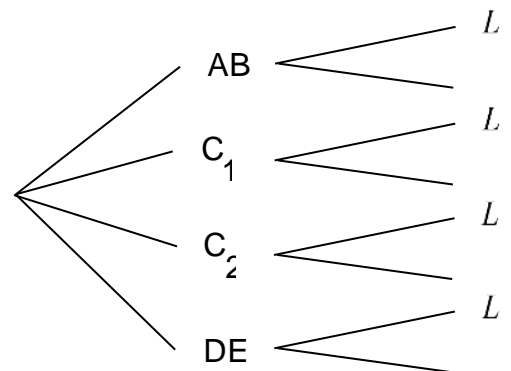
1) Reproduce and complete the probability tree using all the above data.
2) Give the probability that the respondent is a skilled manual worker voting for Remain.
3) Give the probability that the respondent is a leave voter.
4) If the respondent decides to vote for Leave, what is the probability that he belongs to class A or B ?
5) *Subsidiary question :*
   The union of classes A, B and C1, called ABC1, is often (but wrongly) identified as the middle class and the class C2DE as the working class. If the respondent decides to vote for Leave, what is the probability that he belongs to the middle class?
   You may use the fact that: $(AB \cup C_1) \cap L = (AB \cap L) \cup (C_1 \cap L)$.

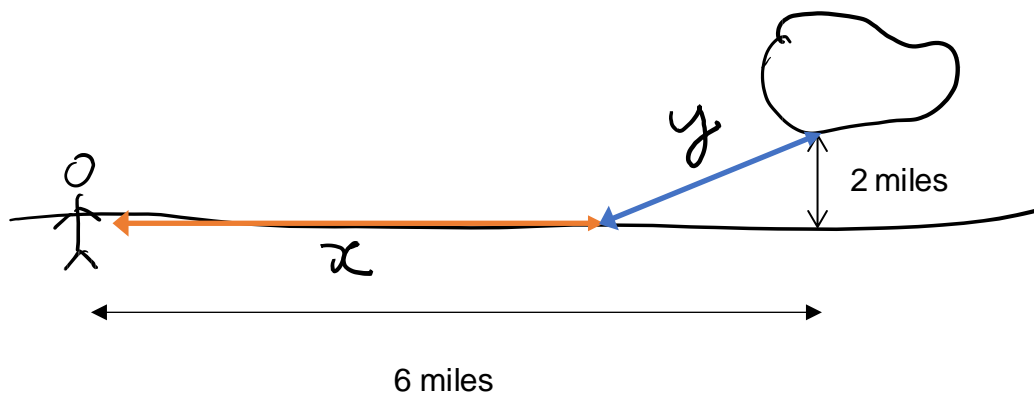**Vocabulary:**   Respondent: enquêté(e)

Sujet n°10

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Function

An island is 2 miles away perpendicular from the shoreline

Tom is staying 6 miles away along the shoreline. He is planning to go to the island by running and then swimming.

Let's assume that Tom runs at a rate of 8mph and swims at a rate of 3mph.

6 miles

You now have to determine how long Tom should run before swimming to minimize the time it takes to reach the island.

Let's assume Tom runs x miles along the shoreline and then swims y miles to reach the island.

1) Show that the total time to reach the island depending on x is $T(x) = \frac{x}{8} + \frac{\sqrt{x^2 - 12x + 40}}{3}$.

2) Determine the derivative function T' of the function and its sign.

3) Prove that Tom needs to run approximately 5.2 miles before swimming to minimize the time to reach the island.

Sujet n°11

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

Here is a picture of four models numbered (1), (2), (3) and (4). Some of the cubes are hidden behind other cubes.

(1)   (2)   (3)   (4)
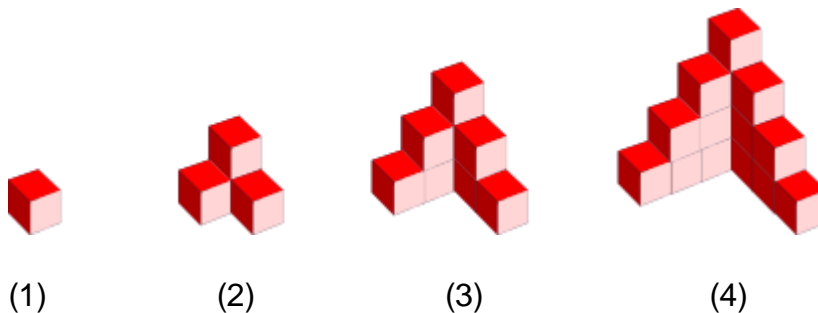
Model (1) consists of one cube.

Model (2) consists of four cubes and so on.

We label u_n the needed number of cubes to build the model (n). So $u_1 = 1$ and $u_2 = 4$.

1) How many cubes are in the model (3) or what is u_3 ?
2) How many cubes are in the model (4) or what is u_4 ?
3) If a model (5) was built, how many cubes would it take or what is u_5 ?
4) Compute the difference between two consecutive terms : $u_2 - u_1$; $u_3 - u_2$ and $u_4 - u_3$
   Deduce an expression of $u_{n+1} - u_n$ in terms of n.
5) Find an expression for the number of cubes used in the nth model.

Sujet n°12

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Probabilities

In York, it's rainy one third of the days.

Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$.

If it's rainy and there is heavy traffic, Jenny arrives late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it is not rainy and there is no heavy traffic.

In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25.

You pick a random day.

1) Model this situation with a probability tree.

2) What is the probability that it's not raining and there is heavy traffic and Jenny is not late ?

3) Show that the probability that Jenny is late is $\frac{11}{48}$.

4) Given that Jenny arrived late at work, what is the probability that it rained that day ?

Sujet n°13

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Functions

Naïm would like to watch some video-on-demand. He is comparing 3 offers from a provider :

- Offer #1 : a monthly fee of €13.99 for an unlimited number of videos.
- Offer #2 : a monthly fee of €5 plus €0.30 per watched video.
- Offer #3 : No monthly fee but €0.70 per watched video.

Reminder : "A monthly fee" is a fixed sum of money you have to pay each month just to access the service.

What offer should Naïm choose ?

Sujet n°14

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

In 2019, the population of codfish offshore a coastline was estimated to 5 000 tons. Because of the overfishing, this population has significantly reduced close to this coastline.

In order to prevent the total extinction of this species, local authorities have decided to control and limit the fishing of codfish.

We suppose that, without the fishing factor, the population of codfish would stay constant at 5 000 tons.

In 2019, the quota of fishing for codfish under this coastline was fixed at 600 tons,

Let's note $U(0) = 600$.

Local authorities have decided to drop this quota by 30 tons each year.

a) Calculate the quota of codfish, expressed in tons, that can be fished in 2020, called $U(1)$.

b) Calculate the quota of codfish, expressed in tons, that can be fished in 2021, called $U(2)$.

c) In a general way, we call $U(n)$ the quota of codfish, expressed in tons, that can be fished during the year $(2019 + n)$.
   How do we call a sequence such as $U(n)$? Give its common difference.

d) Compute $U(10)$ and give an interpretation of this result in the context presented above.

e) Using your calculator, give the total quantity of codfish fished between 2019 and 2029 included.

f) Is this new regulation efficient enough to prevent the total extinction of codfish in this area?

(codfish = cabillaud)

Reminder : for any integer $n \geq 1 : \sum_{k=0}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.
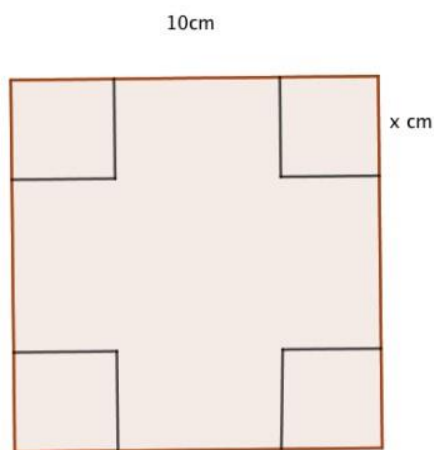
Sujet n°15

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Functions

The side of a square cardboard is 10cm long.

Zoe decides to cut a little square in each corner to fold its sides in order to obtain a jewelry box. The side of the little square is called x.

10cm

x cm

1) What are the possible values for x?

2) Determine the volume $V(x)$ of the jewelry box.

3) Let's call $V'(x)$ the derivative of $V(x)$.

   Calculate $V'(x)$ and check that $V'(5) = 0$.

4) Study the sign of $V'(x)$.

5) Determine the variations of V.

6) Zoe would like her jewelry box to be as big as possible. What value of x should she choose?

Sujet n°16

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

In a school, on the 1st of January, started an outbreak of flu.

This first day just one student was ill, then on the January 2nd there were four new cases, then on January 3rd six new cases, on January 4th seven new cases.

The head of school decided to model that epidemic, to see what would happen.

She created three sequences, as follows:

- (u_n) is the sequence of that gives the number of new cases in terms of the date
- (d_n) is the sequence such that for any integer $n \geq 2$: $d_n = u_n - u_{n-1}$
- (Δ_n) is the sequence such that for any integer $n \geq 3$: $\Delta_n = d_n - d_{n-1}$

1) Copy and fill the following table:

n	u_n	d_n	Δ_n
1	1		
2	4		
3	6		
4	7		

2-a) You should notice something about (Δ_n). What is it?

2-b) Assuming that what you noticed about (Δ_n) holds for any integer $n \geq 3$, add new lines to your table, and fill it in as long as u_n stays strictly positive.

2-c) How many students will have suffered from this flu from the 1st of January to the end of the epidemic?

3) The explicit formula for (u_n) is $u_n = a \times n^2 + b \times n + c$ where a; b and c are parameters we are going to determine.

    a) Show that for any integer $n \geq 2$: $d_n = a(2n - 1) + b$

    b) For any integer $n \geq 3$: express Δ_n in terms of n

    c) Then using the numbers in your table, determine a; b and c.

Sujet n°17

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Linear equations

Mr Smith who is florist buys 302 flowers to make corsages* for the Winter Dance.

A carnation corsage used three flowers and a rose corsage uses two flowers.

A carnation corsage sells for $11, while a rose corsage sells for $20.

Rudolf Ernst
The Flower Vendor
Source : Wikipedia

1. The florist wants to optimize his production. Write simultaneous equations involving the number c of carnation corsages and the number r of rose corsages he should use.

2. How many flowers of each type should the florist use in order to maximize gross sales?

3. What amount will he get from the total sale?

4. He then sold all the carnation corsages and 20 rose corsages. What discount should he allow on the remaining rose corsages to sell the rest for $210 ?

* corsage : *petit bouquet*

Sujet n°18

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

The technicians of an aquaculture farm want to set up the automatic dispenser of a chemical product to improve the quality of the water in a tank. The optimal concentration of the product, expressed in $mg.l^{-1}$ (milligrams per litre), is between 140 and 180 $mg.l^{-1}$.

At the beginning of the test, the concentration of the product in the tank is 160 $mg.l^{-1}$.

They estimate that the concentration decreases by 10% each week.

In order to follow the recommendations on the concentration of the product, they plan to program the dispenser to add a certain quantity of product each week.

They try to calculate it so that:

- The concentration of the product follows the recommendations without their intervention during 6 weeks;

- The quantity of consumed product is minimal.

They first try to set up the dispenser to add 10 $mg.l^{-1}$ per week.

We observe the evolution of the concentration each week. This situation can be modelled by a sequence (C_n), where C_n represents the concentration of the product, in $mg.l^{-1}$, at the beginning of the nth week.

Hence $C_0 = 160$.

1. Justify that for any integer n, $C_{n+1} = 0.9 \times C_n + 10$.

2. We label (V_n) the sequence that for any integer n is given by: $V_n = C_n - 100$.

    a. Prove that (V_n) is a geometric sequence with common ratio $0,9$ and first term $V_0 = 60$.

    b. Express V_n in terms of n.

    c. Deduce that for any integer n: $C_n = 0.9^n \times 60 + 100$.

3. Determine the limit of sequence (C_n) when n tends to infinity. Justify your answer.

4. Comment on the previous result according to the technicians' expectations.

5. After how many weeks will the concentration be lower than 140 $mg.l^{-1}$?

6. Does this set up correspond to the expectations?

Sujet n°19

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Statistics

The table below is an extract from the monthly report on the labour market in the UK, issued by the UK Office for National Statistics in February 2020.

	Usual weekly hours of work	number of workers (in thousands)
Employees	from 0 up to 6 hours	278
	from 6 up to 15 hours	1,580
	from 16 up to 30 hours	5,196
	from 31 up to 45 hours	15,945
	from 45 up to 55 hours	4,727
Self-employed	from 0 up to 6 hours	147
	from 6 up to 15 hours	448
	from 16 up to 30 hours	1,136
	from 31 up to 45 hours	1,999
	from 45 up to 55 hours	1,297

1) According to this table, how many workers are there in the UK?
2) How is this labour force divided between employees and self-employed workers?
3) Knowing that the UK population is approximately 66.5 million inhabitants, what is the percentage of the working population in this country?
4) What is the median class of this series of data?
5) Compute the mean of usual weekly hours of work:
    a) for employees,
    b) for self-employed workers,
    c) for all workers.

Sujet n°20

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Functions

"Capitals" is a company which manufactures pens.

Its cost of production, in thousands of pounds, is modelled by the function C, defined and differentiable in the interval [0, 60]. C is a function of x, the number of thousands of pens produced, such as $C(x) = \frac{1}{4}(x^2 - 50x + 500)$.

1) What does (0) represent?

2) Let's assume that a pen is sold £2.50 and that all the pens are sold.
   Show that the profit of the company, in thousands of pounds, can be modelled by the function

$$P(x) = -\frac{1}{4}x^2 + 15x - 125$$

   x being the number of pens produced, in thousands.

3) Draw and comment the graph of function P. Use it to find out the approximate number of pens the company has to sell to be profitable.

4) Check your previous answer by computing the exact number of pens for which the company is profitable. What is the maximum profit it can make?

Sujet n°21

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Percentage and Functions

This is a true or false exercise. For each of the 4 statements, you need to say if it is either true or false and justify your answer.

1) The management of a company decides to decrease the budget allowed to the expenses for business travels.

Statement 1: "Decreasing this budget by 6% every year during five consecutive years corresponds to decreasing the budget of 30% over this period of five years".

2) This enterprise produces and sells USB sticks. The monthly production varies between 0 and 10,000 sticks. The monthly benefit, expressed in thousands of euros, can be modelled by function B defined on the interval [0; 10] by the expression:
$$B(x) = -x^2 + 10x - 9,$$
where x represents the number of thousands of USB sticks produced and sold.

Statement 2: "When the enterprise produces and sells between 1000 and 9000 USB sticks, the benefit is positive".

Statement 3: "When the enterprise produces and sells 5000 USB sticks, the benefit reaches its maximum".

Statement 4: "When the enterprise produces and sells between 2000 and 8000 USB sticks, the average monthly benefit is equal to 78,000 euros".

Reminder: The average of function f, defined and continuous on the interval [a; b], is:

$$\frac{1}{b-a} \times \int_a^b f(x)dx.$$

<u>Sujet n°22</u>

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Sequences

In its issue of 1st of June 2019, a magazine proposes a sports plan to be fit for the summer holidays, which consists in walking a certain distance every day according to the following pattern:

On June 1 you have to walk 2000 meters.

Each consecutive day, you have to increase the distance covered the day before by 3% and add 150 more meters.

For any integer n, we define by u_n the distance you have to walk on the nth day of June.

Therefore $u_1 = 2000$ is the first term of this sequence.

1) Compute u_2 and u_3.
2) Justify that for any integer n we have: $u_{n+1} = 1.03 \times u_n + 150$.
3) For any integer n, we define $v_n = u_n + 5000$
   Prove that sequence (v_n) is geometric with common ratio equal to 1.03 and give its first term.
4) Express v_n in terms of n.
5) Deduce that for any integer n, the expression of u_n in terms of n is:
$$u_n = 7000 \times 1.03^n - 5000$$
6) Compute on which day the distance to be walked will exceed 4000 meters.
7) Compute the distance walked in total in the whole month of June.

   Reminder: for $n \geq 0 \; and \; q \neq 1 \; ; S_n = 1 + q + q^2 + \cdots + q^n = \frac{1-q^{n+1}}{1-q}$

Sujet n°23

Please do not write on this document and do not forget to hand it back to the jury at the end of the exam.

Statistics

The ages of participants in a fund-raising walk were distributed as follows:

Age	Frequency	Cumulative frequency
[10; 15[	28	
[15; 20[	65	
[20; 25[	82	
[25; 30[	76	
[30; 35[	54	
[35; 40[	43	
[40; 50[	12	

1) Calculate an estimate of the mean age.
2) Complete the table.
3) Estimate the median, and the upper and lower quartiles.
4) Draw a box plot to represent these data.
5) What other statistics parameters could you use to describe these data ?